

Dynamic calculation of a steel-concrete cylindrical shell by FEM

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Abstract. This paper proposes a method for calculating the natural frequencies of a two-layer cylindrical shell placed in a soil medium. The first method is based on the use of analytical expression obtained using the semi-integral theory of cylindrical shells. The second is based on the method of finite elements (FEM) with the construction of the computational model in the environment Lira Sapr. The modeling of composite shell layers in the software package was carried out by plates connected with each other by means of absolutely rigid bodies. The soil accounting was realized in two ways: 1) by creating an array of volumetric bodies; 2) by setting the paste coefficient for the outer layer of the composite shell. It is noted that the second method of ground accounting allows to reduce the time of data input in 5-6 times with the same results. The discrepancy between the results of the determination of the frequencies of natural vibrations according to the analytical method and the finite element method does not exceed 10%. The use of the analytical expression allows to calculate the results 10 times faster and does not require special software, therefore it is more advantageous when determining the frequencies for a 2-layer cylindrical shell.

1 Introduction

Cylindrical casings laid in a soil medium are typically used in pipeline transportation. The multi-kilometer structure is laid in various soils, including sections with waterlogged soils. Measures against pipe floating in such areas are performed by concrete weighting devices, which can damage the pipe during the robot production process. One of the methods to exclude such situations is the use of composite pipes, the inner part of which is made of large-diameter steel pipes ($d < 1000$ mm) parameterized $0.015 \leq h/R \leq 0.05$, and the outer part is covered with a 4-10 cm thick concrete layer. Designers are faced with the question of which calculation method to use when determining the natural vibration frequencies of these structures.

In the published sources for the last 10 years, an approach using analytical formulas, as well as the finite element method in various software packages is proposed. For example, in [1, 2], analytical formulas are used to determine the frequencies of natural vibrations of isotropic pipelines, taking into account the velocity of fluid flow, which were obtained for the calculation scheme in the form of a rod. This approach does not allow to take into

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account the deformation of the pipe section and can be used for thick-walled pipes with parameters $0.07 < h/R < 0.125$. In [3] the dynamics of an underwater pipeline is investigated, but the internal pressure is not considered. In [4, 5] the influence of internal unsteady pressure is investigated for closed cylindrical shells, but the issue of external environment is not covered. The authors in [6-8] determine the frequencies of natural vibrations using different shell theories: Sodel, Flugge, Morley-Coiter and Donnell. The result of the solution, using these theories, is a determinant, by expanding which the frequency of natural vibrations is determined. In [9, 10] the vibrations are investigated without taking into account the ground, and the solution is obtained using the semi-unmomentum theory of cylindrical shells Vlasov-Novozhilov. In [11], formulas for determining the natural frequencies for large-diameter pipelines partially buried in the ground are derived; in [12], a similar approach is realized for a 2-parameter foundation. In [13], frequencies for a metal-ceramic cylindrical shell placed in an elastic Pasternak base are studied, but the internal pressure is not taken into account. Works [14-16] are devoted to 3-layer shells, but the functional of the obtained solutions is extremely narrow, since it does not take into account the internal pressure on the wall of the shell, as well as the repulsion of the medium, preventing the deformation of the walls. In [17] modeling is performed in ANSYS, [9] in ABAQUS, and further the authors compare the obtained results with the results of calculations using analytical formulas.

The purpose of this paper is to compare the frequencies of natural vibrations for a closed two-layer cylindrical shell placed in an elastic soil medium, obtained using the finite element method and analytical formulae.

2 Methods

The section of cylindrical two-layer shell with the radius of steel layer is investigated $R=710$ mm, thickness $h_2=1.8$ cm. Thickness of concrete lining $h_1=4.0$ cm. The moduli of elasticity of concrete and steel, as well as the density of the layers are assumed to be equal to $E_1= 3.24711 \cdot 10^{10}$ (N/m²), $E_2= 2.06 \cdot 10^{11}$ (N/m²), $\gamma_1= 24516.6$ (N/m³), $\gamma_2= 76982.2$ (N/m³). Poisson's ratio for steel and concrete is assumed to be the same $\nu=0.3$. Ground medium parameters: soil density $\gamma_{gr}= 11770$ N/m³; ground elastic modulus $E_{gr}= 500000$ N/m²; Poisson's ratio of the soil $\nu_{gr}=0.49$. Internal pressure p_0 disregarded.

The Lira Sapr software was used for the determination of frequencies by the first method. The modeling of the layers of the two-layer shell (Fig.1) was performed by plates, and to ensure joint operation of the layers, the displacement unions for each corresponding node were used by means of absolute rigid bodies (ARB). In order to model the hinged immobile supports of the ends of the considered section, the linear displacements of the boundary nodes along the Z and Y axis were constrained.

Soil accounting was performed in two ways. In the first one, a soil array was created from volumetric bodies with the size of 5.3×5.3 m. In the second method, ground conditions were taken into account by the pastel coefficient $C_{1z}= 473620$ (N/m³) for the concrete layer. Of the loads, only the dead weight of the shell layers was taken into account.

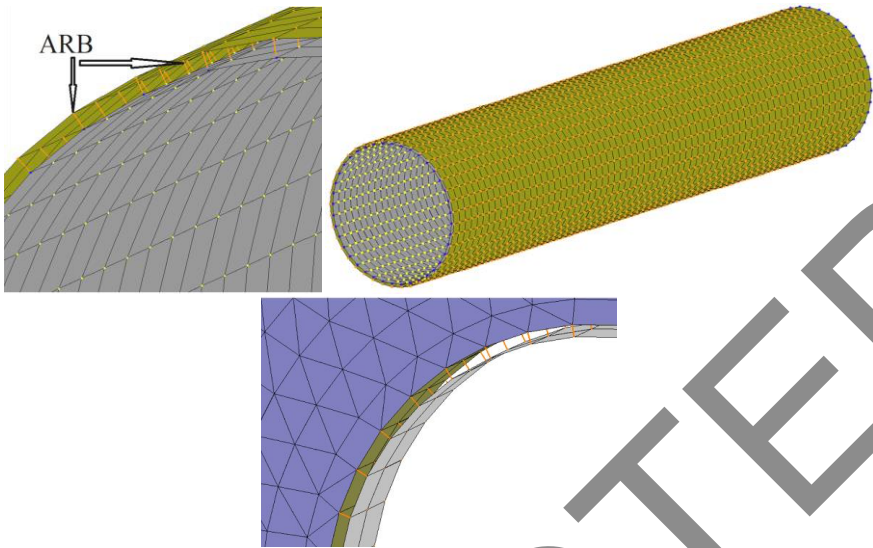


Fig.1. Modeling of a cylindrical shell.

The second method is based on the analytical expression (1), which was obtained in [18] for a two-layer cylindrical shell. The derivation of this expression is based on the semi-integral theory of cylindrical shells for articulated support of the ends of the section under consideration:

$$\omega_{nm} = \frac{1}{2\pi} \sqrt{\frac{\lambda_n^4 + \eta \cdot m^2 (m^2 - 1) \left(m^2 - 1 + \frac{P^*}{\eta} \right) + C_{1z}^* \cdot m^4}{\rho_{sh}^* R_0 \cdot h (\lambda_n^4 h_v + m^4 + m^2)}}, \quad (1)$$

where:

n – number of half-waves in the longitudinal direction;

m – number of half-waves in the circumferential direction;

$\lambda_n = n\pi R_0 / L \sqrt{h_v}$ – length parameter of a two-layer cylindrical shell;

L – section length (m);

$R_0 = R - Z_0$ – shell radius (m);

R – radius of the steel layer of the shell (m);

$Z_0 = \frac{E_1 h_1^2 - E_2 h_2^2}{2(E_1 h_1 + E_2 h_2)}$ – distance from the jointing layer to the original surface (m) (Fig.1);

h_1 – thickness of the concrete layer of the shell (m);

h_2 – thickness of the steel layer of the shell (m);

$h = h_1 + h_2$ – wall thickness of the two-layer shell (m);

E_1 – modulus of elasticity of concrete layer (N/m²);

E_2 – modulus of elasticity of steel layer (N/m²);

$h_v = h / R_0 \sqrt{12(1 - \nu^2)}$ – relative shell thickness parameter;

ν – Poisson's ratio;

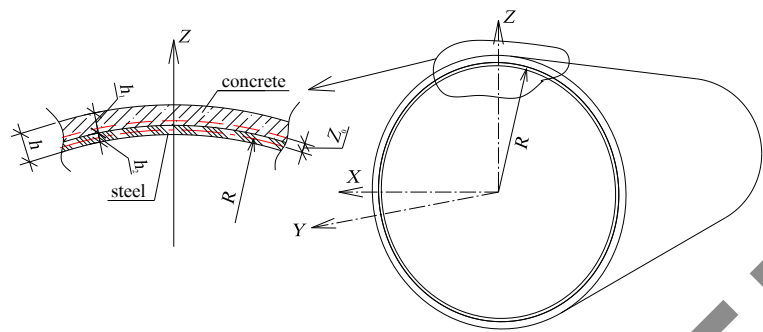


Fig. 2. Geometric dimensions of the two-layer shell.

$\eta = E_v / E_0$ – heterogeneity factor;
 $E_v = (1 - \nu^2) \cdot 12D / h^3$ – reduced modulus of elasticity (bending);
 $D = \frac{1}{3(1 - \nu^2)} [E_1 \{ (h_1 - Z_0)^3 + Z_0^3 \} + E_2 \{ (h_2 + Z_0)^3 - Z_0^3 \}]$ – flexural stiffness;
 $E_0 = [E_1 h_1 + E_2 h_2] / h$ – reduced modulus of elasticity (tensile/compression);
 $p^* = p_0 (R_0 / E_0 h \cdot h_v^2)$ – internal working pressure parameter;
 p_0 – internal pressure in the two-layer shell (N/m²);
 $\rho_{sh}^* = \rho_0 (R_0 / E_0 \cdot h \cdot h_v^2)$ – shell material density parameter (c²/m²);
 $\rho_0 = [(\gamma_1 h_1 + \gamma_2 h_2) / h] / g$ – reduced density of the shell material (N·c²/m⁴);
 γ_1 – concrete density N/m³;
 γ_2 – steel density N/m³;
 $C_{1z}^* = R_0^2 C_{1z} / E_{gr} h \cdot h_v^2$ – reduced pastel coefficient;
 $C_{1z} = E_{gr} / R_0 (1 + \nu_{gr})$ – pastel coefficient for a cylindrical shell (N/m³);
 K_{gr} – ground elastic modulus (N/m²).

3 Results

The calculated frequencies of natural vibrations for different section lengths are summarized in Table 1 for ease of analysis.

Table 1. The results of determining the frequencies of natural oscillations in various ways.

Analytical formula (Hz)	PC Lira Sapr array (Hz)	PC Lira Sapr ratio C1z (Hz)	Analytical formula (Hz)	PC Lira Sapr array (Hz)	PC Lira Sapr ratio C1z (Hz)	Analytical formula (Hz)	PC Lira Sapr array (Hz)	PC Lira Sapr ratio C1z (Hz)
1	2	3	1	2	3	1	2	3
L=7 m (R/L=1/10)			L=8 m (R/L=1/11)			L=9 m (R/L=1/13)		
$\omega_{11}=71.5$	$\omega_{11}=69.8$	$\omega_{11}=68.9$	$\omega_{11}=55.1$	$\omega_{11}=57.0$	$\omega_{11}=56.0$	$\omega_{11}=43.8$	$\omega_{11}=48.4$	$\omega_{11}=47.14$
$\omega_{12}=64.2$	$\omega_{12}=69.9$	$\omega_{12}=63.1$	$\omega_{12}=62.5$	$\omega_{12}=68.2$	$\omega_{12}=67.5$	$\omega_{12}=61.6$	$\omega_{12}=67.2$	$\omega_{12}=66.5$
$\omega_{13}=169.4$	$\omega_{13}=164.7$	$\omega_{13}=164.4$	$\omega_{13}=169.3$	$\omega_{13}=164.1$	$\omega_{13}=163.8$	$\omega_{13}=169.2$	$\omega_{13}=163.8$	$\omega_{13}=163.5$
$\omega_{22}=109.2$	$\omega_{22}=109.6$	$\omega_{22}=109.3$	$\omega_{22}=92.1$	$\omega_{22}=95.4$	$\omega_{22}=95.0$	$\omega_{22}=81.6$	$\omega_{22}=86.2$	$\omega_{22}=79.2$
$\omega_{23}=174.3$	$\omega_{23}=174.6$	$\omega_{23}=174.3$	$\omega_{23}=172.1$	$\omega_{23}=170.9$	$\omega_{23}=170.6$	$\omega_{23}=170.9$	$\omega_{23}=168.7$	$\omega_{23}=168.4$
$\omega_{32}=211.9$	$\omega_{32}=186.5$	$\omega_{32}=186.3$	$\omega_{32}=167.4$	$\omega_{32}=154.8$	$\omega_{32}=154.6$	$\omega_{32}=137.6$	$\omega_{32}=132.3$	$\omega_{32}=132.0$
$\omega_{33}=194.6$	$\omega_{33}=199.7$	$\omega_{33}=199.5$	$\omega_{33}=184.4$	$\omega_{33}=188.2$	$\omega_{33}=187.9$	$\omega_{33}=178.7$	$\omega_{33}=181.3$	$\omega_{33}=180.7$

The first column shows the calculation results using expression (1), the second column shows the results for the FEM when the soil medium array is created, the third column shows the results for the FEM when the bed coefficient is set, the results correspond to the seven vibration forms, the images of which are shown in Figure 3. For clarity, Figure 4 shows the first 3 forms of oscillations in the soil array.

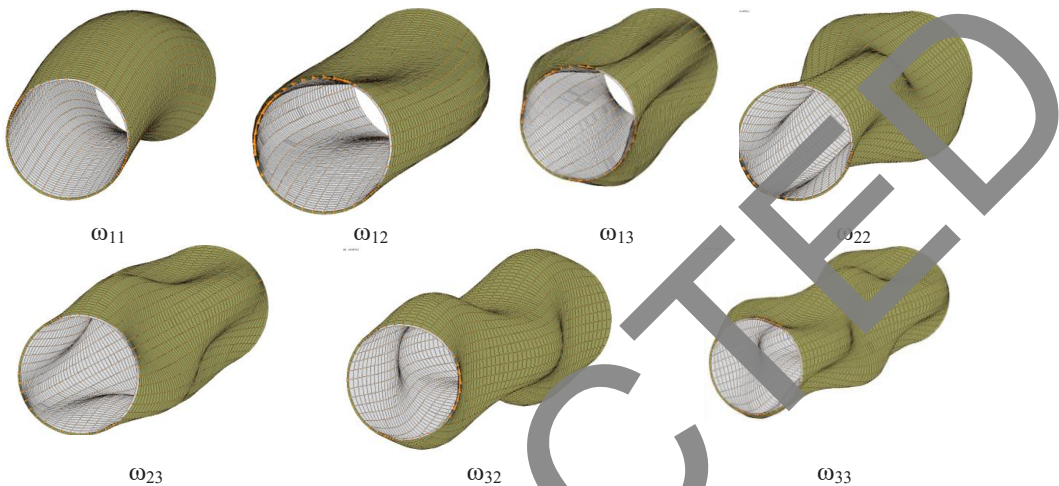


Fig. 3. Waveforms for the pipeline section under consideration

The analysis of the results recorded in Table 1 shows that the difference in the frequency values determined by FEM with the ground array assignment (column 2) and by assigning the pastel coefficient (column 3) does not exceed 2%, therefore, to reduce the labor input during model creation, it is recommended to use the second method of taking into account ground conditions, which will also reduce the time of data processing by the processor by 5-6 times.

Comparison of the results of determination of natural vibration frequencies using expression (4) and FEM shows that the difference for the first 3 frequencies does not exceed 6% and for the rest of the results 10%.

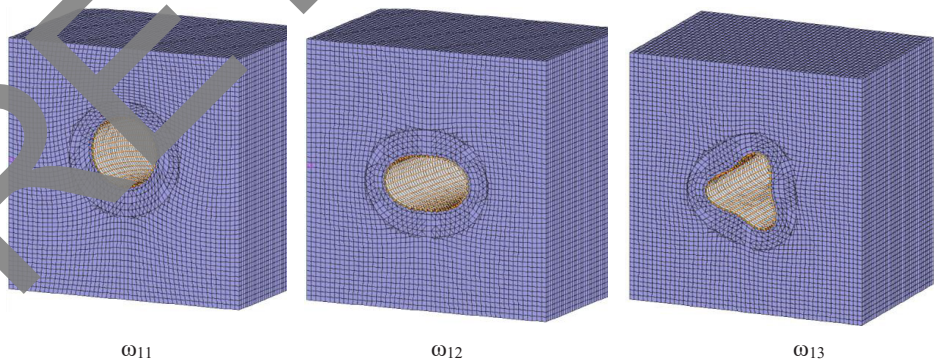


Fig. 4. The waveforms for the pipeline section under consideration at $n=1$ in the soil mass (the size of the array is 5.3x5.3 m in cross section).

The analytical method has clear advantages over the finite element method, since the calculation of frequencies took 10 times less time with practically the same results. Also,

using the analytical method, it is possible to take into account the influence of internal working pressure by the parameter p^* , while using MFE, this factor cannot be taken into account, because when modeling this loading, it is taken into account not as a force preventing the deformation of the cross section, but as an additional mass taken into account when calculating the frequencies, so all the data in Table 1 are obtained with zero internal pressure.

4 Conclusion

1. The discrepancy between the frequencies of natural vibrations for the object of study determined by the analytical method and FEM does not exceed 10%, and for the first 3 frequencies of the spectrum does not exceed 6%, therefore, all methods are applicable. However, the use of analytical expression allows to calculate the results 10 times faster and does not require specialized software, so it is more advantageous when tuning the design by frequency characteristics.

2. When calculating natural vibration frequencies by the finite element method, the second method of setting ground conditions allows reducing the time of data input by 5-6 times with the same results.

3. On the basis of the analysis carried out for the purpose of the greatest productivity it is recommended to use the analytical expression given in this paper when designing large diameter pipeline transportation facilities.

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