Decomposition method in the study load-loading forestry manipulator mechanism mathematical model

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Abstract. In this article a new hydrokinematic diagram of the boom lifting mechanism with an energy recuperation system in starting and braking modes is presented. The mathematical model of the lifting mechanism of a forest manipulator with a hydro-mechanical damper taking into account the rigidityof the metal structure of the boom and the hydraulic drive, described by a system of nonlinear differential equations of the second order, has been developed. In linearised formulation it is reduced to a second order differential equation with operator coefficients. The methods of operator theory and functional analysis are used to solve the problem. Reducing the dynamicloading and energy intensity of the working processes of the boom lifting mechanism of the forestry manipulator by justifying the parameters of the energy- saving damping device of the hydraulic drive allows increasing the reliability of the manipulator, reducing energy costs, as well as well as the downtime for repairs due to the failure of hydraulic equipment. The results obtained can be used in the design of recuperative systems for other lifting equipment in construction and agriculture.

1 Introduction

In [1] the working processes of the forwarder manipulator were investigated:under different loading cycles depending on the load volume and the number of assortments. It was found that the time of movement with load is 63%, 30% of time is spent on idle moves, 7% of time is spent on operations inside the load compartment. There is a high dynamic load of technological equipment in forest conditions.

In works [2-3] results of researches of operating modes of electrohydrostatic drive of hydraulic manipulators with load weight 30 000 kg, manometric pressure of preliminary charging of the hydraulic accumulator 5 bar are given. The efficiency and advantages over traditional valve-controlled drives are shown.

In work [4] with application of mathematical and simulation modelling the research of working modes of regenerative hydraulic drive of a timber truck- manipulator is carried out, it is established that pressure pulsations do not exceed 0.4 MPa.

In works [5-6] studies of dynamic characteristics of the joint motion of the boom and arm of hydraulic manipulators have been carried out. In these modes, increased fluctuations of

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operating pressure at the nominal level of 12.5 MPa are observed, with the angular velocity of the arm exceeding the velocity of the more loaded boom.

In the article [7] on the basis of theoretical studies of the dynamics of the forest manipulator lifting mechanism, the influence of the coefficient of pliabilityof flexible elements of the hydraulic drive on the dynamic loading of the structure was established.

In [8], a method of increasing the reliability of metal structures of lifting mechanisms of manipulators by damping the maximum pressure fluctuations of the working fluid in hydraulic drives is proposed.

However, the research of dynamic and kinematic characteristics of energy- saving devices as applied to the lifting mechanisms of forest manipulators, in our opinion, has been carried out insufficiently, therefore, carrying out additional research in this direction is relevant**.**

The aim of the research is to reduce the dynamic load and energy consumption of working processes of the boom lifting mechanism of the forest manipulator by justifying the parameters of the energy-saving hydraulic drivebased on the solution of the mathematical model.

2 Materials and Methods

In order to study the process of loading and unloading operations of the forest manipulator during forest harvesting, taking into account the connection of an additional damper to the hydraulic drive of the boom lifting mechanism, protected by the patent [9], to reduce dynamic loads and increase reliability, the design schemes of the boom lifting mechanism (Fig. 1) and the damper (Fig. 2) aredrawn up.

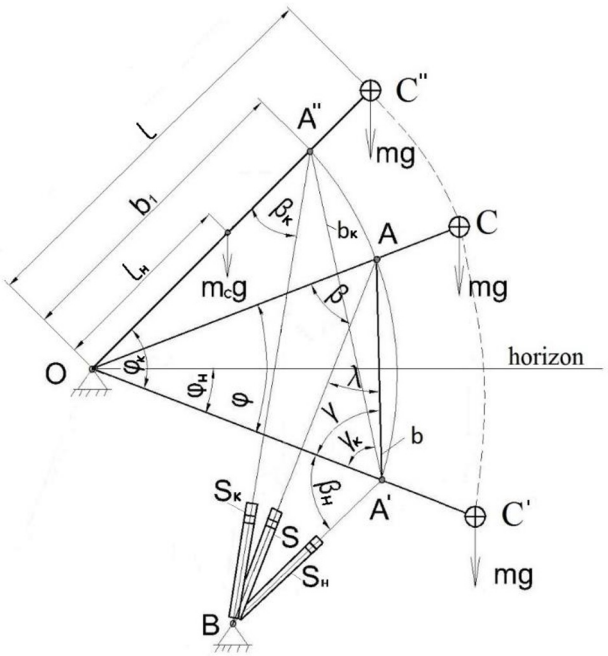

Fig. 1. Calculation diagram of the boom lifting mechanism of the manipulator arm.

The mathematical model of the hoisting mechanism of the forest manipulator with the hydromechanical damper is developed taking into account therigidity of the metal structure of the boom and the hydraulic drive:

$$\begin{cases} \left(J_{\text{бр}} + J_{\text{c}}\right)\frac{d^2\varphi}{dt^2} = \frac{\pi d_c^2 b_1 \sin\beta}{4} p - \left(G_{\text{бр}} L + G_{\text{c}} l_{\text{н}}\right)\cos\varphi - \frac{G_{\text{бр}}\, C_{\text{ст}}\left(L - b_1\right)^4 \cos\varphi}{3EJ}, \\ q_{\text{н}} n_{\text{н}} = \frac{\pi d_c^2 b_1 \sin\beta}{4} \cdot \frac{d\varphi}{dt} + \mu \frac{\pi d_{\text{п}}^2}{4} \sqrt{\frac{2p}{\rho}} + a_{\text{у}} p + \frac{1}{C_{\text{г}}} \cdot \frac{dp}{dt}, \\ \frac{d^2 x_{\text{д}}}{dt^2} = \frac{1}{m_{\text{д}}}\left(p \frac{\pi D_{\text{д}}^2}{4} - c_{\text{п}} x_{\text{д}} - \frac{dx_{\text{д}}}{dt} k_{\text{д}}\right), \end{cases} \quad (1)$$

where t - time, s;
$p(t)$ - current value of pressure in the pressure pipeline, Pa;
$\varphi(t)$ - angle of boom lift, rad;
$x_{\text{d}}\ (t)$ - stroke of the damper plunger, m;
t_{k} - time for lifting process, s;
p_{n} - initial value of pressure p, Pa;
φ_{n} - initial value of the angle φ, м; $\varphi_{\text{к}}$ - final value of the ngle φ, м;
x_{dn} - is the initial position of the ram stroke, m;
x_{dk} - end position of the ram stroke, m;
S_{n} - minimum length of the hydraulic cylinder, m;$S_{\text{к}}$ - maximum length of the hydraulic cylinder, m;
m_{br} - mass of log bundle, kg;
m_{c} - weight of the boom group, kg;
J_{br} - moment of inertia of the log bundle relative to point O, kg·m^2 ;
J_{c} - moment of inertia of the boom relative to point O, kg·m^2 ;
G_{br} - gravity of the log bundle in the gripper, N;
G_{c} - gravity of the boom, N;
d_{c} - inner diameter of the boom hydraulic cylinder, m;
b_1 - parameter, m
L - outreach of the manipulator;
l_{n} - distance from point O to the centre of mass of the boom group;
q n_{nn} - feed rate per unit of time, m^3 /c;μ is the flow coefficient;
d_{p} - throttle orifice diameter, m;
ρ - density of working fluid, kg/m^3;
a_{y} - leakage coefficient in the hydraulic system, m^3 /(Pa/c);
c_{g} - volumetric stiffness of the hydraulic drive N/m^3;
m_{d} - mass of damper plunger, kg;
d_{d} - diameter of damper plug, m;
c_{p} - stiffness of damper spring, N/m;
k_{d} - coefficient of friction of the damper plug;
C_{st} - stiffness of the boom metal structure, N/m;
E - modulus of elasticity of the boom, N/cm^2;
J - moment of inertia of the boom cross section, m^4

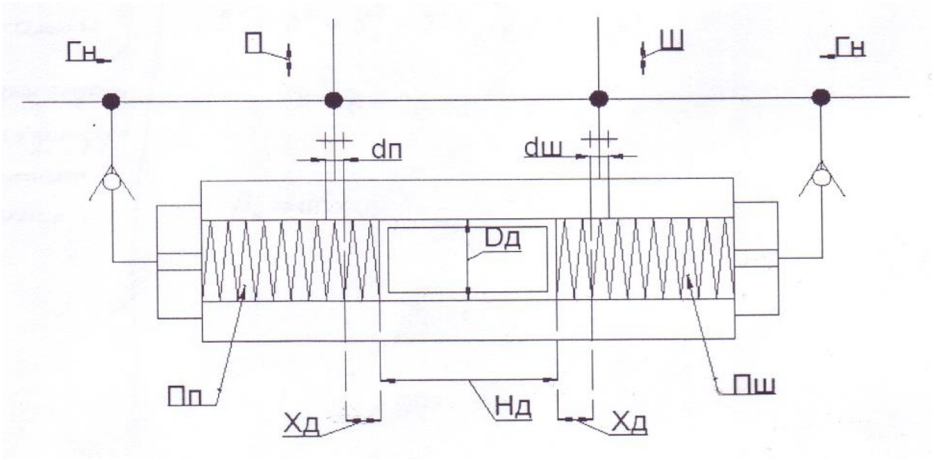

Fig. 2. Calculation diagram of the hydromechanical damper

The angle β is related to the angle φ by the following expressions (see Fig.1)

$$\beta = \gamma - \lambda, \quad \lambda = \arcsin\left(\frac{S_n}{S}\sin(\beta_n + \gamma)\right)$$

$$S^2 = b^2 + S_н^2 - 2bS_{st}\cos(\beta_н + \gamma), \quad b = 2b_1\sin\left(\frac{\varphi}{2}\right)$$

$$\beta_n = \arccos\left(\frac{b_k^2 + S_n^2 - S_k^2}{2b_k S_n} - \gamma_k\right), \quad b_k = 2b_1\sin\left(\frac{\varphi_k}{2}\right)$$

$$\gamma_к = \frac{\pi}{2} - \frac{\varphi_к}{2}, \quad \gamma = \frac{\pi}{2} - \frac{\varphi}{2}$$

For the system (1.1) we set the boundary value problem on the interval [0; $t_к$]:

$$p(0) = p_n$$
$$\varphi(0) = \varphi_n, \ \varphi(t_k) = \varphi_k \qquad (2)$$
$$x_d(0) = x_{d_n}, \ x_d(t_k) = x_{d_k}$$

Note 1. The requirement of differentiability of the function (t) entails thenecessity to take the condition p_n non-zero. This is also consistent with physical sense, as otherwise an undesirable vacuum is created. To avoid this, the oil is fedinto the pump at a low head; the tank is positioned above the pump.

In order to solve the problem, the system of equations (1) is linearised by assuming that the expression $\sin\beta$ not depends on t. This is acceptable because the value of the expression $\frac{\pi d_c^2 b_1 \sin\beta}{4}$ is negligibly small (it takes values of the order of 10^{-2} modulo; its influence on the solution of the problem is also negligibly small − for example, there is no boundary layer phenomenon near the initial point).

To solve the problem, the segment [0; t_k] is divided by n equally spaced points t_i with step $h = \frac{t_k}{n}$; and the required functions are defined at these points.

The system of differential equations (1) is replaced by a system of recurrence relations at the highest term. Thus, the sought functions are found in the form of recurrence relations at the nodal points of the time variable. Due to the fact that the matrix at the eldest term is degenerate, the decomposition method is used for the resolution.

Let us replace in (2) the boundary conditions at the point $t=t_к$ by the initial conditions at the point $t = 0$. For this purpose, let us decompose the desired function $\varphi(t)$ decompose in the neighbourhood of the initial point by the formula $\varphi(t) = \varphi(0) + t\varphi'(0)$ and, taking $t=t_к$, we obtain the initial condition for finding the function $\varphi(t)$:

$$\varphi'(0) = \frac{\varphi_k - \varphi_n}{t_к} \tag{3}$$

$$x_d{}'(0) = \frac{x_{d_k} - x_{d_n}}{t_k} \tag{4}$$

Similarly, we obtain the initial condition for finding the $x_d(t)$:

3 Research results

Consider the initial problem:

$$\mathcal{A}u_{i+2} = \mathcal{B}u_{i+1} + \mathcal{C}u_i + \mathcal{F}_i \tag{5}$$

$$u_0 = u^0, \ u_1 = u^1 \tag{6}$$

where $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ — matrices; u^0, u^1, $\mathcal{F}_i$ are given; $i = 0,1,2,....$ It is assumed that $\mathcal{F}_i$ is defined at each $i = 0,1,2,....$

Matrix $\mathcal{A}$ is degenerate because of det $\mathcal{A}$=0. It is known that the operator given by a degenerate square matrix is fredholms [10] with zero index (hereafter, fredholms).

Here $\mathcal{A}$ possesses a one-dimensional kernel.

Here is the necessary information.

For the matrix $\mathcal{A}$ let's introduce the projector $\mathcal{Q}$ on Coker $\mathcal{A}$, the semi-inverse matrix $\mathcal{A}^- = \tilde{\mathcal{A}}^{-1}(\mathcal{I} - \mathcal{Q})$ ($\tilde{\mathcal{A}}$ denotes the contraction of the matrix $\mathcal{A}$ on Coim $\mathcal{A}$; $\mathcal{I}$ – unit matrix in the corresponding subspace).

Let us fix a non-zero element $z \in$ Ker $\mathcal{A}$, an element $\psi \in$ Coker $\mathcal{A}$. Coker $\mathcal{A}$ define the scalar product $< , >$ such that $< \psi , \psi > = 1$.

Let us proceed to the solution of the problem (7), (8).

Let us solve equation (7) with respect to u_{i+2}. Application of the resultsobtained in [11] and the decomposition method leads to the following theorem

Theorem 1. Let the following condition be

$$< \mathcal{Q}\mathcal{B}z, \psi > \neq 0 \tag{7}$$

Then the relation (7) is equivalent to the system

$$u_{i+2}\mathcal{T}_{01}u_{i+1} + \mathcal{T}_{00}u_i + \mathcal{F}_{0i} \tag{8}$$

$$< \mathcal{Q}Bu_{i+1}, \psi > + < \mathcal{Q}\mathcal{C}u_i, \psi > + < \mathcal{Q}\mathcal{F}_i, \psi > = 0 \tag{9}$$

as labelled

$$\mathcal{T}_{01}(\cdot) = \mathcal{A}^-\mathcal{B}(\cdot) - \frac{< (\mathcal{Q}\mathcal{B}\mathcal{A}^-\mathcal{B} + \mathcal{Q}\mathcal{C})(\cdot), \psi >}{< \mathcal{Q}\mathcal{B}z, \psi >} z$$

$$\mathcal{T}_{00}(\cdot) = \mathcal{A}^-\mathcal{C}(\cdot) - \frac{< \mathcal{Q}\mathcal{B}\mathcal{A}^-\mathcal{C}(\cdot), \psi >}{< \mathcal{Q}\mathcal{B}z, \psi >} z$$

$$\mathcal{F}_{0i} = A^-\mathcal{F}_i - \frac{< \mathcal{Q}\mathcal{B}\mathcal{A}^-\mathcal{F}_i + \mathcal{Q}\mathcal{F}_{i+1}, \psi >}{< \mathcal{Q}\mathcal{B}z, \psi >} z$$

Now consider the initial problem (5), (6). The following result holds for it.

Theorem 2: Let condition (7) are satisfied. Then the solution of the problem (5), (6) exists if the matching condition is fulfilled

$$< \mathcal{QB}u^1, \psi > + < \mathcal{QC}u^0, \psi > + < \mathcal{QF}_0, \psi > = 0 \quad (10)$$

Let us proceed to the solution of problem (1)-(4).

Denoting $p_i = p(t_i)$, $\varphi_i = \varphi(t_i)$, $x_{di} = x_d(t_i)$, we replace the derivatives by their difference analoguesat the t_i:

$$\frac{dp}{dt}(t_i) \approx \frac{p_{i+1} - p_i}{h}, \quad \frac{d\varphi}{dt}(t_i) \approx \frac{\varphi_{i+1} - \varphi_i}{h}, \quad \frac{dx_д}{dt}(t_i) \approx \frac{x_{д_{i+1}} - x_{д_i}}{h}$$

$$\frac{d^2p}{dt^2}(t_i) \approx \frac{p_{i+2} - 2p_{i+1} + p_i}{h^2}, \quad \frac{d^2\varphi}{dt^2}(t_i) \approx \frac{\varphi_{i+2} - 2\varphi_{i+1} + \varphi_i}{h^2}$$

$$\frac{d^2x_д}{dt^2}(t_i) \approx \frac{x_{д_{i+2}} - 2x_{д_{i+1}} + x_{д_i}}{h^2}$$

Then the system (1)-(4) in notations

$$\sigma_{11} = J_{бр} + J_c, \quad \sigma_{12} = \frac{\pi d_c^2 b_1 \sin\beta}{4}, \quad \sigma_{13} = -\left[G_{бр}L + G_c l_н + \frac{G_{бр}\, C_{ст}(L - b_1)^4}{3EJ}\right]\cos\varphi_i$$

$$\sigma_{21} = q_н n_н, \quad \sigma_{22} = \mu\frac{\pi d_п^2}{4}\sqrt{\frac{2}{\rho}}, \quad \sigma_{23} = a_у, \quad \sigma_{24} = \frac{1}{Cг}$$

$$\sigma_{31} = \frac{\pi D_д^2}{4m_д}, \quad \sigma_{32} = -\frac{c_п}{m_д}, \quad \sigma_{33} = -\frac{k_д}{m_д}$$

is written in the form (5), (6) with matrices

$$\mathcal{A} = \begin{pmatrix} 0 & \sigma_{11} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} 0 & 2\sigma_{11} & 0 \\ \sigma_{24} & \sigma_{12} & 0 \\ 0 & 0 & 2 + \sigma_{33}h \end{pmatrix}$$

$$\mathcal{C} = \begin{pmatrix} \sigma_{12}h^2 & -\sigma_{11} & 0 \\ -\sigma_{24} + \sigma_{23}h & -\sigma_{12} & 0 \\ \sigma_{31}h^2 & 0 & -1 - \sigma_{33}h + \sigma_{32}h^2 \end{pmatrix}$$

required vector-sequence $u_i = \begin{pmatrix} p_i \\ \varphi_i \\ x_{д_i} \end{pmatrix}$

vector-sequence $\mathcal{F}_i = \begin{pmatrix} \sigma_{13}h^2\cos\varphi_i \\ \sigma_{22}h\sqrt{p_i} - \sigma_{21}h \\ 0 \end{pmatrix}$

initial vector-sequences $u^0 = \begin{pmatrix} p_н \\ \varphi_н \\ x_{дн} \end{pmatrix}$, $u^1 = \begin{pmatrix} p_н + \frac{p_к - p_н}{t_к}h \\ \varphi_н + \frac{\varphi_к - \varphi_н}{t_к}h \\ x_{дн} + \frac{x_{дк} - x_{дн}}{t_к}h \end{pmatrix}$

where all values, except $p_к$, are taken from conditions (2)-(4).

Note 2. The value $p_к$ is used only for solving the problem; it will bedefined further on. This condition is not required in the problem statement.

The matrix P has a one-dimensional kernel. For it we have:

$$z = \begin{pmatrix}1\\0\\0\end{pmatrix},\ \psi = \begin{pmatrix}0\\1\\0\end{pmatrix},\ Q = \begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix},\ \mathcal{A}^- = \begin{pmatrix}0&0&0\\\frac{1}{\sigma_{11}}&0&0\\0&0&1\end{pmatrix}$$

Condition (2.3) is satisfied:

$$< Q\mathcal{B}z, \psi > = \sigma_{24} \neq 0$$

From condition (2.6) follows the expression for finding $p_к$:

$$p_к = \left[1 - \frac{t_к}{h} + \frac{\sigma_{24}-\sigma_{23}h}{\sigma_{24}h}t_к\right]p_н - \frac{\sigma_{22}}{\sigma_{24}}\sqrt{p_н}\,t_к + \left[-\frac{\sigma_{12}}{\sigma_{24}}\right]\varphi_н + \frac{\sigma_{12}}{\sigma_{24}}t_к\varphi_к + \frac{\sigma_{21}}{\sigma_{24}}t_к \quad (11)$$

The recurrence relations for determining the desired functions at the points t_i from system (2.4) follow:

$$p_{i+2} = \frac{\sigma_{24}-\sigma_{23}h}{\sigma_{24}}p_{i+1} - \frac{\sigma_{22}}{\sigma_{24}}h\sqrt{p_{i+1}} - \frac{\sigma_{12}^2}{\sigma_{11}\sigma_{24}}h^2p_i - \frac{\sigma_{12}}{\sigma_{24}}\varphi_{i+1} + \frac{\sigma_{12}}{\sigma_{24}}\varphi_i - \frac{\sigma_{12}\sigma_{13}}{\sigma_{11}\sigma_{24}}h^2\cos\varphi_i$$

$$\varphi_{i+2} = 2\varphi_{i+1} - \varphi_i + \frac{\sigma_{13}}{\sigma_{11}}h^2\cos\varphi_i + \frac{\sigma_{12}}{\sigma_{11}}h^2p_i$$

$$x_{д_{i+2}} = (2+\sigma_{33}h)x_{д_{i+1}} + (-1-\sigma_{33}h+\sigma_{32}h^2)x_{д_i} + \sigma_{31}h^2p_i$$

$$p_0 = p_н,\ \ p_1 = p_н + \frac{p_к - p_н}{t_к}h$$

$$\varphi_0 = \varphi_н,\ \ \varphi_1 = \varphi_н + \frac{\varphi_к - \varphi_н}{t_к}h$$

$$x_{д_0} = x_{д_н},\ \ x_{д_1} = x_{д_н} + \frac{x_{д_к} - x_{д_н}}{t_к}h$$

where $p_к$ is determined by formula (11).
The following graphs are obtained at $t_к$=9.

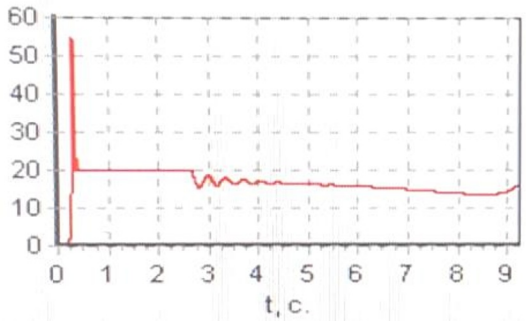

Fig. 3. Graph of pressure dependence on time

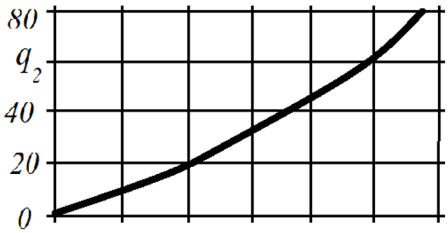

Fig. 4. Graph of boom lift angle dependence on time

4 Conclusion

1. A new forest manipulator lifting mechanism with a damping device to reduce dynamic loads and improve reliability is proposed and protected by a patentfor invention.
2. The mathematical model of the lifting mechanism of a forest manipulator with a hydromechanical damper is developed, taking into account the rigidity ofthe metal structure of the boom and the hydraulic drive, described by a system of nonlinear differential equations of the second order.
3. In the linearised formulation the system of nonlinear differential equationsof the second order is reduced to a differential equation of the second order with operator coefficients.

The methods of operator theory and functional analysis are applied to solve the problem. The sought functions are found in the form of polynomials in powers of time and their graphs are plotted.

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