Synthesis of spatial five- and six-letter mechanisms with rotational pairs by the movement of the output link

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Abstract. The article discusses spatial mechanisms with rotational pairs that allow the movement of the output link or working body with a given accuracy while ensuring the necessary rigidity and reliability of the structure. Its durability is also significantly increased, energy losses are reduced, since all hinge assemblies (rotational kinematic pairs) are made on rolling bearings and are reliably isolated from the external environment. The article is also devoted to the synthesis (design) of functional spatial six-branch mechanisms with rotational pairs reproducing with a given accuracy the linear function of the change in the angle of rotation of the output link, provided it is fully rotatable. The objective function and dependencies necessary for the design of such mechanisms are obtained and an algorithm for solving this problem in CAD is developed.

1 Introduction

Spatial mechanisms are quite widely used in agricultural engineering [1,2]. Basically, four-, five- and six-pin mechanisms are used in the structures, but there are very few of them whose hinges are made in the form of rotational kinematic pairs alone, despite their obvious advantages [3-5,12-15]. Spatial mechanisms with rotational pairs allow the movement of the output link or working body with a given accuracy while ensuring the necessary rigidity and reliability of the structure. Its durability is also significantly increased, energy losses are reduced, since all hinge assemblies (rotational kinematic pairs) are made on rolling bearings and are reliably isolated from the external environment. The tasks of synthesizing such mechanisms, despite their complexity, are undoubtedly important in the theory of mechanisms and machines.

2 Materials and methods

To synthesize spatial 6R mechanisms according to a given law of the output link, an analysis of structural schemes was carried out, the constructive implementation of which will accurately reproduce the required linear motion function. We showed that the reproduction of a linear function on some part of the turnover of the output link can be achieved using two-crank six-link spatial mechanisms (Figure 1a) or their five-link modification (Figure 1b). The structural parameters of these mechanisms are related by dependencies:

$$\alpha_2 = 180^0 - 2\alpha_6, \quad l_2 = 0; \tag{1a}$$

$$\alpha_3 = \alpha_5 - \alpha_1, l_3 = l_5 - l_1; \tag{1b}$$

$$\alpha_4 = 180^0 - \alpha_6, l_4 = l_6; \tag{1c}$$

$$\frac{l_1}{\sin\alpha_1} = \frac{l_5}{\sin\alpha_5} = \frac{l_6}{\sin\alpha_6}. \tag{1d}$$

Here and further, α_i is the angle of intersection of the geometric axes of the hinges of the link, counted counter clockwise, taking the axis of the hinge facing the observer as the starting point; l_i is the shortest distance between the axes of the hinges (the theoretical length of the link).

3 Results and discussion

The kinematics of the output link of spatial mechanisms with rotational pairs depends only on the angular parameters of the links [6]; therefore, they will be the determined parameters when designing mechanisms.

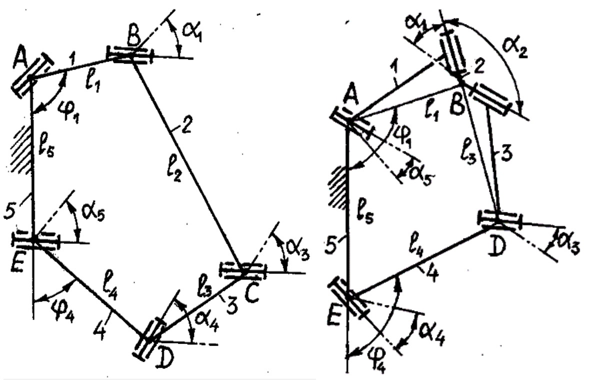
As for the linear parameters, when determining them, it is necessary to set the length of one of the links arbitrarily, based on the functional purpose of the mechanism and the design of the hinge assemblies, while other lengths are determined by the above formulas.

Let's first find the relationship between the rotation angles of the input (φ_1) and output (ψ_5) links of the mechanism. This dependency is defined by the following expression:

$$\psi_5 = \arcsin\frac{(\cos\alpha_6 - \cos\alpha_1)\sin\varphi_1}{1 - \cos\alpha_1\cos\alpha_6 - \sin\alpha_1\sin\alpha_6\cos\varphi_1} + ¿$$

$$+\arcsin\frac{(\cos\alpha_6+\cos\alpha_1)\sin\varphi_1}{1+\cos\alpha_1\cos\alpha_6-\sin\alpha_1\sin\alpha_6\cos\varphi_1}+\text{¿}$$
$$+\arcsin\frac{-(\cos\alpha_6+\cos\alpha_5)\sin\varphi_1}{1+\cos\alpha_5\cos\alpha_6-\sin\alpha_5\sin\alpha_6\cos\varphi_1}. \quad (2)$$

Here α_6 is the angle of intersection of the axes of the hinges of the frame (rack), varying from 85^0 to 65^0 with an interval of 0.5^0. The angle α_5 of crossing the axes of the hinges of the driven link - crank is taken in the range from 213^0-α_6 to 183^0-α_6 with the same interval, and the angle α_1 of crossing the axes of the hinges of the driving crank should be less than ninety degrees.



(a) (b)

Fig. 1. Kinematic schemes of spatial hexagons (a) with the dependence of the angles of intersection of the geometric axes of the joints of the links α_4= 〖180〗^0-α_6; (b) a five-link, with one spherical kinematic pair, modification of the hexagon.

A graphical representation of this expression is given in Figure 2 in two coordinate systems - $\varphi_1 0\psi_5$ and $\varphi C\psi$. Moreover, the second coordinate system is more convenient for carrying out further calculations and programming them on a computer.

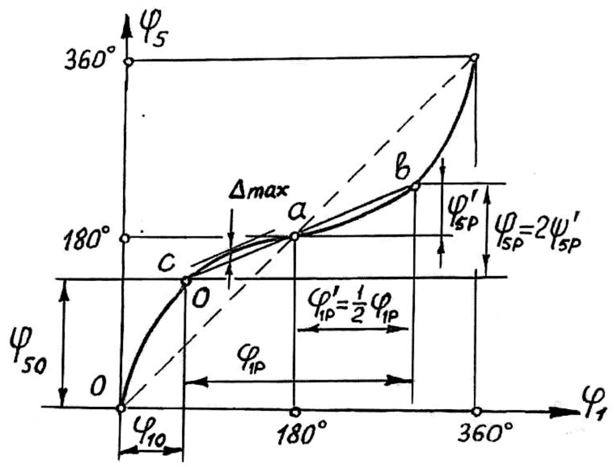


Fig. 2. Graph of the dependence of the angle of rotation of the output link on the angle of rotation of the input for the hexagon.

As can be seen from the above graph, the curve $\psi_5=f(\varphi_1)$ is symmetric with respect to point *a*. Therefore, the straight line segment *cb* = 2*ab* , as well as the angle $\varphi_p=2\varphi_p'$ $(\psi_p=2\psi)$. Preliminary calculations determined that the value of the φ_p of the working angle of rotation of the input crank should be within 180^0 - 300^0. Then the corresponding working angle φ_p of the output crank rotation will be with it in the following relationship [7, 8]:

$$K=\frac{\psi_p}{\varphi_p}=\frac{\psi_p'}{\varphi_p'}=0{,}1\ldots0{,}5. \quad (3)$$

The synthesis of the mechanism is carried out so that the maximum deviation Δ_{max} of the curve $\psi_5=f(\varphi_1)$ in the section *cb* does not exceed the value limited by the relative error:

$$\Delta\psi^0=\frac{\Delta_{max}}{\psi_p}\cdot100\,\%\le\Delta\psi. \quad (4)$$

It is desirable that the deviation be $\Delta\psi=0{,}2-0{,}5\,\%$.

The synthesis of the mechanism will be carried out by the method of the purposeful search from the minimization condition of the function Δ_{max} (or $\Delta\psi^0$) [9]. However, before proceeding to the definition of this function, we will make an equation linking the angle α_1 and other input and output parameters of the synthesis mechanism. It has the form:

$$\alpha_1=\arcsin\frac{A}{B}, \quad (5)$$

where:

$$A=\sin\alpha_6\left[\left(1-\cos\frac{\varphi_p}{2}\cos\frac{K\varphi_p}{2}\right)(1++\cos\alpha_5\cos\alpha_6)+\sin\frac{\varphi}{2}\right.$$
(6)

$$B=\left(\cos\frac{K\varphi_p}{2}-\cos\frac{\varphi_p}{2}\right)\left(1+\cos\alpha_1\cos\alpha_6++\sin\alpha_1\sin\alpha_6\,co\right.$$
(7)

Expression (5) is derived from the equation:

$$l_1\cos\varphi_1=l_6+l_5\cos\varphi_5-\text{¿}\,l_6(\cos\varphi_1\cos\varphi_5--\cos\alpha_5\sin\varphi_1\sin$$
$$\cdot\{[\cos\alpha_5\sin\varphi_1\cos\gamma_3-(\sin\alpha_5\sin\alpha_6--\cos\alpha_5\cos\alpha_6\cos\varphi_1$$
(8)

considering that:

$$\frac{l_1}{\sin\alpha_1}=\frac{l_5}{\sin\alpha_5}=\frac{l_6}{\sin\alpha_6},$$

$$\varphi_1=180^0+\varphi_p^{'}=180^0+\frac{\varphi_p}{2},$$

$$\psi_5=180^0+\psi_p^{'}=180^0+\frac{K\varphi_p}{2}$$

and

$$\sin\gamma_3=\frac{-(\cos\alpha_6-\cos\alpha_5)\sin\varphi_1}{1+\cos\alpha_5\cos\alpha_6-\sin\alpha_5\sin\alpha_6\cos\varphi_1},$$

$$\cos\gamma_3=\frac{(1+\cos\alpha_5\cos\alpha_6)\cos\varphi_1-\sin\alpha_5\sin\alpha_6}{1+\cos\alpha_5\cos\alpha_6-\sin\alpha_5\sin\alpha_6\cos\varphi_1}.$$

In turn, the dependence (8) is found by the solution of the vector equation compiled for the vector contour ABCD (see Figure 3):

$$\overline{AB}=\overline{AF}+\overline{FE}+\overline{ED}+\overline{DC} \qquad (9)$$

in the projection on the Y axis.

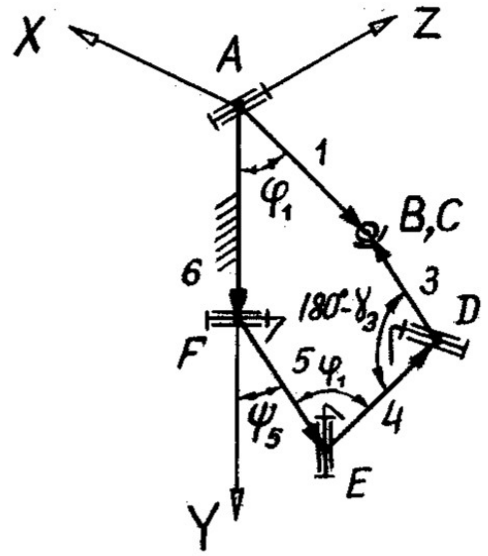


Fig. 3. Kinematic scheme of spatial hexagons with a vector contour AFEDBC.

Now let's move on to the Δ_{max} function. For the mechanism under consideration, it is as follows:

$$\Delta_{max}=\arcsin\frac{(\cos\alpha_6-\cos\alpha_1)\sin\varphi_K}{1-\cos\alpha_1\cos\alpha_6-\sin\alpha_1\sin\alpha_6\cos\varphi_K}+$$

$$+\arcsin\frac{(\cos\alpha_6+\cos\alpha_1)\sin\varphi_K}{1+\cos\alpha_1\cos\alpha_6-\sin\alpha_1\sin\alpha_6\cos\varphi_{1K}}+¿$$

$$-\arcsin\frac{(\cos\alpha_6+\cos\alpha_5)\sin\varphi_{1K}}{1+\cos\alpha_5\cos\alpha_6-\sin\alpha_5\sin\alpha_6\cos\varphi_{1K}}-K\varphi$$

(10)

The angle φ_K included here is found from the equality of the partial derivative (∂ψ_5)/(∂φ_1) to the angular coefficient K, that is:

$$\frac{\partial\psi_5}{\partial\varphi_1}=\frac{\cos\alpha_6-\cos\alpha_1}{1-\cos\alpha_1\cos\alpha_6-\sin\alpha_1\sin\alpha_6\cos\varphi_K^{'}}+¿$$

$$\frac{+\cos\alpha_6+\cos\alpha_1}{1+\cos\alpha_1\cos\alpha_6-\sin\alpha_1\sin\alpha_6\cos\varphi_K^{'}}-¿$$

$$\frac{-\cos\alpha_6-\cos\alpha_5}{1+\cos\alpha_5\cos\alpha_6-\sin\alpha_5\sin\alpha_6\cos\varphi_K^{'}}=K, \qquad (11)$$

where $\varphi_K^{'}=180^0+\varphi_K$

From this expression, after a series of transformations, we get:

$$x^3+ax^2+bx+c=0, \qquad (12)$$

where $x=\cos\varphi_K$.

$$a=\frac{1}{K\sin\alpha_1\sin\alpha_5\sin\alpha_6}\{\sin\alpha_1(\cos\alpha_6+\cos\alpha_5)--2\sin\alpha_5\text{co}$$

(13a)

$$b=\frac{1}{K\sin^2\alpha_1\sin^2\alpha_6\sin\alpha_5}\{2\sin\alpha_1(\cos\alpha_5\sin^2\alpha_6--\sin\alpha_1\text{sir}$$

$$+K[2\sin\alpha_1(1+\cos\alpha_1\cos\alpha_6)+\sin\alpha_5(1--\cos^2\alpha_1\cos^2\alpha_6)]\},$$

(13b)

$$c=\frac{1}{K\sin^2\alpha_1\sin^3\alpha_6\sin\alpha_5}\{(1+\cos\alpha_1\cos\alpha_6)[(1-\cos^2\alpha_1\text{cos}$$

$$\cdot(\cos\alpha_5+\cos^2\alpha_1\cos\alpha_6)\}. \qquad (13c)$$

To solve the cubic equation (12), we find the discriminant:

$$\Delta=\left(\frac{q}{2}\right)^2+\left(\frac{pq}{3}\right)^3, \qquad (14)$$

here

$$q=\frac{2a^3}{27}-\frac{ab}{3}+c, \qquad (15a)$$

$$p=\frac{-a^2}{3}+¿b. \qquad (15b)$$

If the discriminant Δ>0, then the real root of the equation is one:

$$x_1=\sqrt[3]{\frac{-q}{2}+\sqrt{\Delta}}+\sqrt[3]{\frac{-q}{2}-\sqrt{\Delta}}-\frac{a}{3} \qquad (16)$$

and, therefore,

$$\varphi_K=\arccos x_1.$$

If Δ = 0, and *p* and *q* are not equal to zero, then there are two real roots:

$$x_1=\frac{3a}{p}-\frac{a}{3},\ x_2=-\left(\frac{3q}{2p}+\frac{a}{3}\right) \tag{17}$$

and

$$\varphi_{(1)K}=\arccos x_1,\ \varphi_{(2)K}=\arccos x_2.$$

If Δ<0, then there are three real roots:

$$x_1=2\left|\sqrt[3]{r}\right|\cos\frac{\beta}{3}-\frac{a}{3},$$

$$x_2=2\left|\sqrt[3]{r}\right|\cos\frac{\beta+360^0}{3}-\frac{a}{3}, \tag{18}$$

$$x_3=2\left|\sqrt[3]{r}\right|\cos\frac{\beta+720^0}{3}-\frac{a}{3}.$$

Here:

$$\left|\sqrt[3]{r}\right|=\left|\sqrt{\frac{-p}{3}}\right|;\ cos\beta=\frac{-q}{2r};\ r=\left|\sqrt{\frac{-p^3}{27}}\right|.$$

Then:

$$\varphi_{(1)K}=\arccos x_1,$$
$$\varphi_{(2)K}=\arccos x_2,$$
$$¿\,\varphi_{(3)K}=\arccos x_3.$$

The value of the angle φ_K is chosen so that the condition is met $\varphi_K \le \varphi_p^{'}$.

Thus, the dependences necessary for the optimization synthesis by the objective function of a six-link mechanism (or its five-link modification), reproducing a linear function on some part of the output link turnover with its full turn ability, are obtained [10, 11].

Synthesis is carried out as follows. The working section φ_p of the angle of rotation of the input crank is set, as well as the extreme upper values of the coefficient K and the angles α_5 and α_6.After finding the angle α_1, the objective function $\Delta\psi^0$ is calculated, which is compared with the lower value $\Delta\psi$. In this case, the condition must be met:

$$\Delta\psi^0 \le \Delta\psi \tag{19a}$$

or

$$\Delta\psi_{i+1}^0-\Delta\psi_i^0 \ge 0. \tag{19b}$$

If this is not done, then the values of the angles α_5 and α_6 are successively reduced by a certain amount and the process is repeated until one of the conditions (19) occurs. If all possible parameter values are K, α_5, α_6 are sorted out, and the condition $\Delta\psi^0 \le \Delta\psi$ is not obtained, then $\Delta\psi$ increases by a certain amount and the process repeats until the condition (19a) is reached.

Having determined the values K, α_1, α_5 and α_6 with the corresponding value $\Delta\psi$, they proceed to determine other parameters of the mechanism.

The value of the working (rectilinear) section of the law of change of the angle of rotation of the output crank:

$$\psi_p=K\varphi_p. \tag{20}$$

The origin of the coordinates of the new system φCψ is determined by the coordinates (see Figure 2):

$$\psi_{50}=180^0-\frac{\psi_p}{2},\ \varphi_{10}=180^0-\frac{\varphi_p}{2}. \tag{21}$$

The angles α_2,α_3 and α_4, as well as the linear parameters of the links, for a given length of one of them, are determined by formulas (1).

4 Conclusion

Optimal structural schemes of spatial multi-link mechanisms with rotation-al pairs (as well as their five-link modification with a spherical kinematic pair) have been identified, which reproduce the required accuracy, the linear function during the rotational movement of the output link.

For these mechanisms, the objective function and the corresponding analytical dependencies are obtained, which can be used to automate the design of such spatial mechanisms in CAD.

In addition to the objective function, additional equations linking the desired parameters and design data have been compiled. The boundaries of their change have been established, which makes it possible to simplify the task of purposefully searching for parameters of the designed mechanisms.

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