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A comparison of generalized extreme value, gumbel, and log-pearson distributions for the development of intensity duration frequency curves. A case study in Costa Rica

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Abstract. Global warming has already affected frequency and intensity of extreme rainfall events. This makes the evaluation of current and alternative statistical distributions used in the formulation of Intensity Duration Frequency curves (IDF) curves highly relevant. This study aims to evaluate the suitability of applying the Generalized Extreme Value (GEV) and the Log-Pearson type 3 (LP3) probability distributions against the traditionally used Gumbel (EV1) distribution to derive IDF curves for a flood prone area located in northern Costa Rica. A ranking system based on a normalized total-score from five metrics was implemented to identify the best distribution. GEV proved to be the most suitable distribution for most storm-durations and was therefore selected for development of the IDF curves with return periods ranging from 2 to 100 years. As return periods get longer however, deviations between rainfall estimates obtained get more prominent. Hence, a meticulous analysis of adjustment to select the most adequate probability distribution to estimate extreme events with return periods of 50 years or more should be undertaken, regardless of GEV or any other distribution. Results also reinforce the need to identify the distribution that best fits observed data for a particular weather station, especially when time-series are asymmetric.

1 Introduction

The purpose of fitting a dataset to various statistical distributions is to be able to estimate the probability of occurrence of extreme precipitation intensities for a given return period [1]. Thus, the maximum amount of precipitation for a given storm duration is calculated and then converted to an intensity. This intensity value is necessary for many

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design calculations, most commonly to determine maximum flow or maximum runoff in order to estimate design floods in the planning and management of infrastructure such as roads, culverts and bridges [2]. The estimated return values are needed to construct Intensity Duration Frequency curves (IDFs), which are widely used in engineering applications. These curves show the relationship between precipitation intensity and storm duration for a given return period. IDF curves are developed for specific locations with specified return periods. IDF curves are developed using historical observed data with the assumption that the same underlying processes will govern future rainfall patterns and resulting IDF curves. Global warming however, has already affected frequency and intensity of extreme rainfall events, which makes the evaluation of current and alternative statistical distributions of paramount importance [3]. Costa Rica, located in the Central America region has been identified as one of the main emerging tropical hot-spot due to increasing variability in precipitation patterns as a consequence of climate change [4]. Consequently, this study aims to evaluate the suitability of applying the Generalized Extreme Value (GEV) and the Log-Pearson type 3 (LP3) probability distributions against the traditionally used Gumbel (EV1) distribution to derive IDF curves for a flood prone area located in northern Costa Rica. It is forecasted that the developed methodology could be extended and applied to other locations across the country.

2 Methodology

2.1 Study area

Costa Rica is located along the Central American isthmus, bordering to the west with the Pacific Ocean and with the Caribbean Sea to the east (Figure 1b). This favors the climatic influences of both oceans over the entire territory which is meriodionally divided by a north-west-south-east mountain-range of high-complexity slopes (Figure 1a). Climate variability is characterized by interactions between local topography and a combination of various climatic drivers. Consequently, six independent climatic regions have been identified (Figure 1a) in which the north-east and south-west domains are determined by the position and elevation of the aforementioned mountain-range [5].

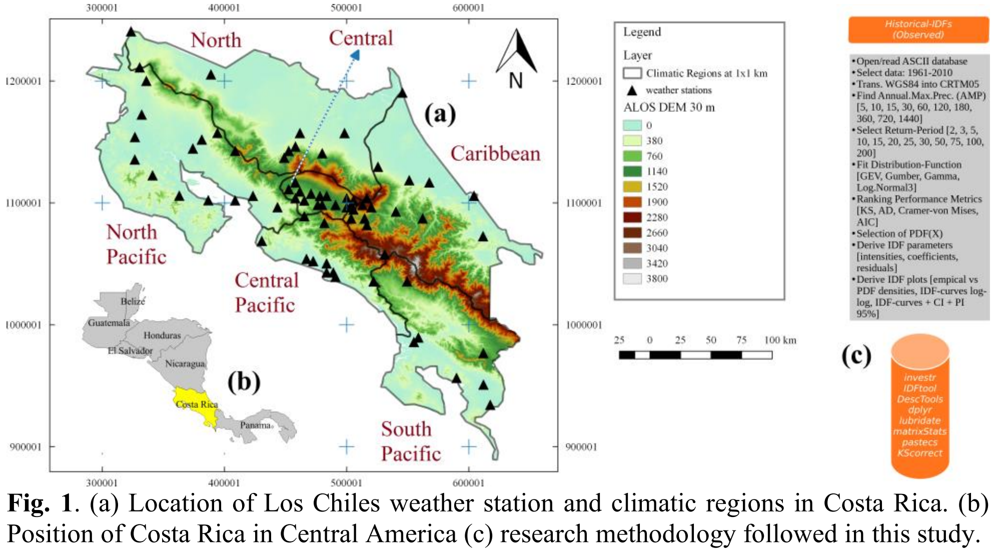

Fig. 1. (a) Location of Los Chiles weather station and climatic regions in Costa Rica. (b) Position of Costa Rica in Central America (c) research methodology followed in this study.

2.2 Research methodology and data sources

The research methodology followed in this study (Figure 1c) consists of: (1) depuration and quality control of observational datasets provided by Instituto Meteorológico of Costa Rica (IMN); (2) spatial data processing and derivation of annual maximum precipitation (AMP) for each duration; (3) parameter optimization and performance assessment of each distribution and (4) selection of best distribution and generation of IDF curves based on global ranking system. One minute precipitation data from Los Chiles weather station (Long: -84.71, Lat: 11.04) for a period of 29 years (1994-2022) were used in this study. The 1-min historical records were aggregated to durations of 5, 10, 15, 30, 60, 120, 180, 360, 720 and 1440 minutes to determine AMPs for each time-step. The station was selected on the basis of its geographical location in the North climatic region of the country along the Costa Rica-Nicaragua border (Figure 1a), which in recent years has coincided with the trajectory followed by several hurricanes and tropical storms and is prone to considerable flooding [6]. All data processing was executed using the R programming language [7].

2.3 Statistical distributions tested

The current IMN (Costa Rica) recommended distribution is Gumbel (EV1) [8], however, as new studies emerge and the necessity to find the most appropriate distributions increases, the Generalized Extreme Value (GEV) used in Canada, and the Log-Pearson type 3 (LP3) used in the USA were also included in this study, as these distributions have been found to often exhibit a better performance when compared to EV1 [9]. It is worth mentioning that both LP3 and GEV are 3-parameter distributions, whereas EV1 only uses 2 parameters only.

2.4 Goodness-of-fit and distribution selection

Goodness-of-fit tests are widely used in statistics to assist finding the best distribution to use in fitting a given dataset. The Anderson-Darling (AD) [critical value = *2.5018*], Kolmogorov-Smirnov (KS) [critical value = *0.20517*], and Cramer-von Mises (CVM) [critical value = *0.22101*] tests were used to compare the goodness of fit between distributions set at 5% significance level ($\alpha = 0.05$). Additionally, the Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC) were also included. To select the best distribution, a global ranking system based on the total score obtained by all metrics was implemented. Each metric was assigned a value ranging from 1 to 3, with the best distribution receiving the lowest value. Thus, the final global ranking for each distribution tested ranges from 5 to 15. For instance, the best distribution is the one with the lowest ranking value.

2.5 Intensity duration frequency curves

Intensity Duration Frequency curves (IDFs) were generated considering return periods of 2, 3, 5, 10, 15, 20, 25, 30, 50, 75 and 100 years. Nonlinear regression model empirical-coefficients for the best distribution were calibrated by the Levenberg-Marquardt algorithm and evaluated by means of the coefficient of determination (R2) and the root mean squared error (RMSE) objective functions. The resulting model equation representing every IDF for each return period is derived as follows:

$$I\,(\mathrm{D}) = \mathrm{A} / (\mathrm{B}+\mathrm{D})^{\mathrm{C}} \tag{1}$$

Where: I are intensities (mm/h) per each return period (years), D is time duration (min) and A, B and C are the empirical-coefficients.

3 Results and discussion

3.1 Statistics of annual maximum precipitation (AMP)

From historical data, AMP for each storm-duration was calculated (Table 1). Descriptive statistics show heavy rainfall extremes for short-duration (5 to 30 min), with intensities ranging from 186 to 101 mm/hr, which are typical of the North climatic region of Costa Rica [10]. Most of these events have been recorded from September to November, which coincides with the period of increased activity in terms of hurricanes and tropical cyclones in the Atlantic Ocean [11]. Datasets exhibiting skewness values ranging from -0.5 to 0.5 are considered symmetric as compared to a normal distribution which exhibits zero skewness. Experimental skewness ranged from -0.517 (30 min) to 0.346 (120 min), which suggest a highly symmetrical dataset for most durations with the exception of 30 min, which experimental value (-0.517) is marginally superior to 0.5. The coefficient of variation (CV) ranged from 0.246 (5 min) to 0.409 (1440 min) with an average value of 0.338, which clearly indicates both the non-normality of the dataset and the high variability around the mean of each duration, particularly for daily totals (1440 min) which reaches 0.409.

Table 1. Statistical parameters of annual maximum precipitation (AMP) for all durations.

Duration (min)	Mean (mm/hr)	Highest (mm/hr)	Lowest (mm/hr)	CV	Skewness
5	118.159	186.000	50.400	0.246	0.268
10	103.227	160.020	33.600	0.260	-0.190
15	92.221	145.280	22.400	0.283	-0.392
30	67.456	101.000	12.400	0.314	-0.517
60	40.556	72.900	6.200	0.375	0.178
120	24.444	51.100	3.100	0.409	0.346
180	18.205	35.000	2.067	0.379	0.096
360	9.825	17.567	1.033	0.359	-0.024
720	5.465	9.675	0.600	0.355	-0.081
1440	3.230	5.808	0.458	0.402	0.320

3.2 Performance of distributions tested

Regarding goodness-of-fit tests, there was only one rejection by KS corresponding to the Gumbel distribution for duration 15 minutes, where the experimental value (0.2169) is slightly higher than the adopted critical value (0.20517). Normalized KS values on the other hand, favors EV1 over the GEV or LP3 distributions for durations 5 and 1440 minutes respectively, both of which exhibit slightly larger skewness values but still below the theoretical asymmetry coefficient for the EV1 distribution (1.1396), which could indicate some sensitivity towards higher extremes (Figure 2). As the KS test is based solely on the greatest difference between observed and estimated frequencies, an outlier could imply the rejection of the null hypothesis, since KS assumes that the distribution to be tested is previously known.

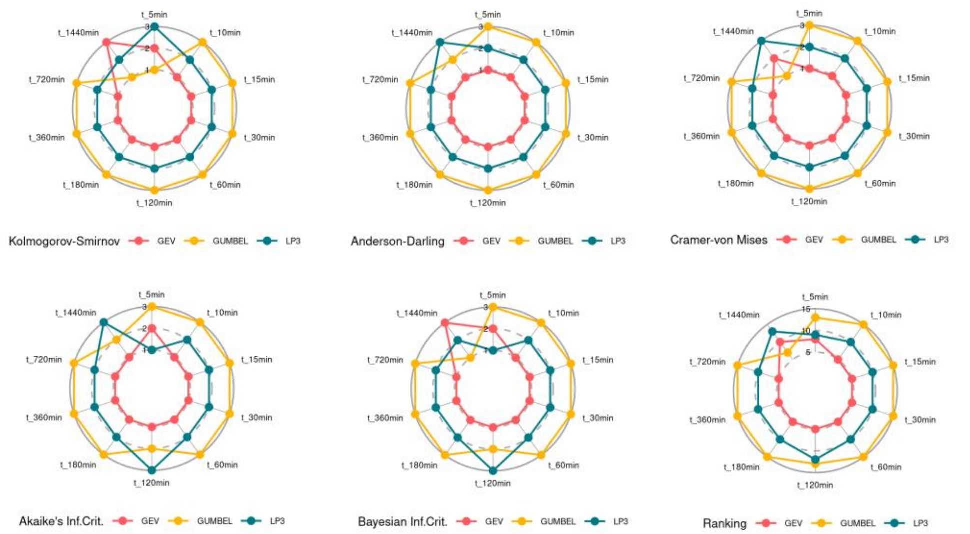

Fig. 2. Normalized goodness-of-fit metrics and global-ranking results for all durations considered.

This is not always the case in practical application as adjusted distribution parameters are usually based on the series to be tested [12]. Nevertheless, KS indicates a better performance of GEV for the remaining durations, with LP3 generally scoring second. On the other hand, no rejections for AD or CVM were detected, which proves that these tests are more rigorous as compared to KS and therefore more appropriate to evaluate the adequacy of probability models for the presently analysed datasets. In addition, the simultaneous use of several metrics, clearly demonstrates confidence in the selection of the best probability distribution as compared to use only one objective function. Normalized AD and CVM values (Figure 2) position GEV as the most suitable distribution except for duration 1440 min, which once again is better represented by EV1. It is worth noticing that for daily durations (1440 min), the Gumbel (EV1) distribution seems to better represent series with positive skewness with high values. The AIC and BIC criteria show substantial agreement with the other metrics (Figure 2). This shows that the different metrics used in the selection of distribution may privilege different probability distributions according to the various durations considered and all criteria contribute proportionally to the general score. As a whole, the results also indicate that any of the three distributions tested (EV1, GEV or LP3) could be used to fit data from the Los Chiles weather station, except for one KS marginal case. This is to be expected, since skewness values are highly symmetrical for all durations observed, which coincides with an almost null rejection rate (Table 1). Nonetheless, in the global ranking, GEV was found to be the best distribution for all durations except 1440 min, where the general consensus is the EV1 distribution is the most suitable, with GEV scoring second and LP3 scoring third. Additionally, the shape parameter (Kappa) of the GEV distribution was estimated for each storm-duration. The Kappa parameter determines the shape of the GEV distribution, were values close to 0 are desirable in practical applications, since it would indicate that the distribution is neither upper bounded nor lower bounded [13]. Higher Kappa values produce minimal differences in magnitudes between large return periods (100 years or more). Experimental Kappa values range from 0.0848 to 0.4750, with an average value of 0.2635, further supporting the suitability of the GEV for the accounted dataset in most cases.

3.3 Generation of intensity duration frequency curves

Generated IDF curves show the precipitation intensity (mm/hour) for each storm-duration for return periods ranging from 2 to 100 years (Figure 3). As aforementioned, results hereby presented are based on GEV calculations only. As anticipated, intensity of rainfall decreases with increases in storm duration. Furthermore, a rainfall of any given duration will have a larger intensity if its return period gets larger. For the 5-min duration, the minimum value is 115.38 mm/hr, and the maximum is 206 mm/hr at the 2-year and 100-year return periods respectively.

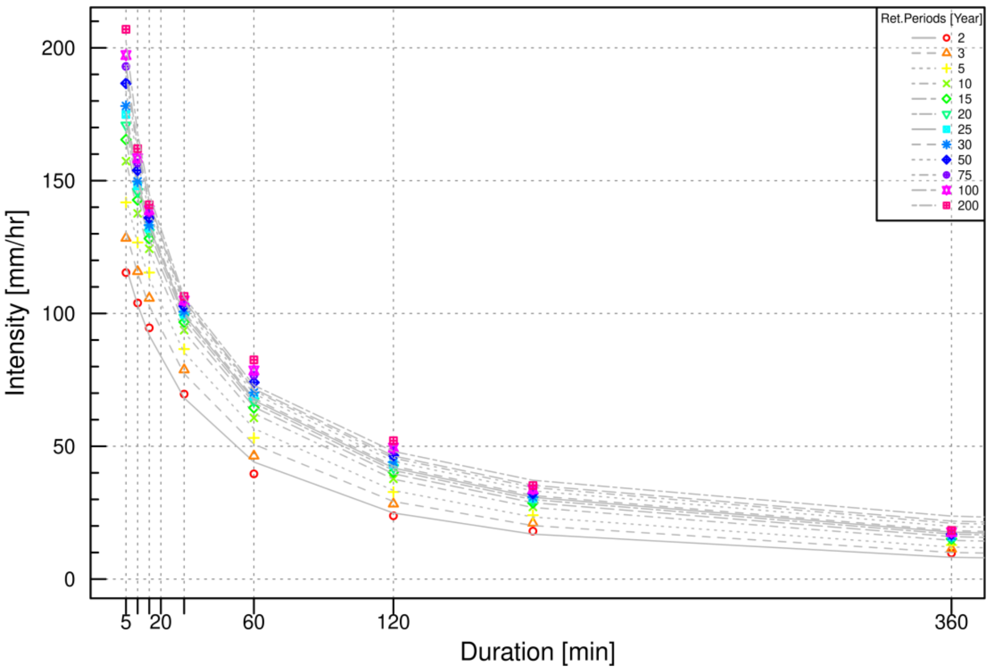

Fig. 3. Intensity Duration Frequency curves (IDF) obtained using the GEV distribution.

The minimum and maximum values for the 1440-min (24-hour) storm duration are 3.07 mm/hr and 7.52 mm/hr correspondingly. Rainfall extremes of such magnitudes emerge first in shorter durations and frequencies. As previously discussed, short-duration rainfall extremes in Costa Rica are part of large-scale circulation systems, with considerable forcing from larger drivers linked to atmospheric circulation patterns over the Central American isthmus. This includes the Western Hemisphere warm pool (WHWP), the seasonal cycle of sea surface temperatures (SSTs) affecting the annual cycle of precipitation, convective activity and storm development and the El Niño-Southern Oscillation (ENSO), which complex warm or wet responses vary in terms of their signs, magnitudes, duration and seasonality [14]. On the other hand, the coefficient of determination (R2) for each return period (Table 2) is notably close to 1, which indicates a strong relationship within the IDF derived equations. As return periods get longer, the deviations between rainfall estimates obtained are more prominent, as exhibited by increasing root mean squared error (RMSE). Hence, a meticulous analysis of adjustment to select the most adequate probability distribution to estimate extreme events with return periods of 50 years or more should be undertaken, regardless of GEV or any other distribution. Estimates of extreme precipitation for longer return periods can significantly differ depending on the distribution model used. This calls into question the level of protection offered by generated IDF curves and whether it is appropriate in locations where observed data clearly demonstrate that alternative probability distributions projections may vary considerably among themselves. This is particularly critical, as heavy rainfall extremes have been shown to be intensifying globally with climate change at a rate generally consistent with the increase in atmospheric moisture, indicating stronger increases in short-duration extreme rainfall intensities than those observed historically [15-16].

Table 2. IDF curves empirical-parameters using the GEV distribution.

Duration (min)	Mean (mm/hr)	Highest (mm/hr)	Lowest (mm/hr)	CV	Skewness
2	11125.6480	38.7049	1.2045	0.9975	2.1214
3	10709.8720	38.9770	1.1648	0.9980	2.0824
5	8347.3950	36.1355	1.0934	0.9990	1.6406
10	5507.4600	30.6784	0.9943	0.9998	0.8009
15	4325.7660	27.3558	0.9398	0.9999	0.6527
20	3668.9630	25.0568	0.9033	0.9997	0.9498
25	3244.5430	23.3299	0.8763	0.9995	1.2899
30	2944.4820	21.9632	0.8551	0.9993	1.5958
50	2282.8330	18.3776	0.7996	0.9984	2.4992
75	1900.8860	15.8114	0.7597	0.9975	3.2218
100	1686.5590	14.1478	0.7337	0.9967	3.7251

4 Conclusions and recommendations

The final selection of a probability distribution for the development of Intensity Duration Frequency curves (IDF) can modify the estimates of extreme events and thus impact the quantification of maximum flows, with intense consequences in hydraulic-works sizing and risk-areas definition. Historical precipitation data from the Los Chiles weather station for the period 1994-2022 has been used in this study to select an appropriate distribution for the estimation and design of IDF curves for storm durations of 5, 10, 15, 30, 60, 120, 180, 360, 720 and 1440 min. The established global ranking system, consisting of several metrics is considered the most suitable approach to select the best probability distribution, superior to the use of just one objective function. The GEV was then selected as the best distribution for most storm-durations except for 1440 min, where the general consensus is the EV1 distribution is the most suitable, with GEV scoring second and LP3 scoring third. The shape parameter of the GEV distribution was also analysed, as values close to zero are desired for practical applications as it ensures that the distribution is not upper-bounded.

However, all adjusted distributions (EV1, GEV or LP3) could be used to fit data from the Los Chiles weather station, except for one KS marginal case and null rejections by AD or CVM. Even when the GEV distribution has shown to be the strongest fitting distribution out of the three tested models when using the Los Chiles dataset, more studies of the application of GEV distribution on other climatic regions of Costa Rica is recommended to ensure its countrywide applicability. Nonetheless, the developed methodology could easily be applied to any other neighbouring station. These results also reinforce the need to detect the most suitable distribution that best fits observed data for a particular weather station, especially when time-series are asymmetric or extreme values are present. Likewise, as return periods get longer, deviations between rainfall estimates obtained with the different probability distributions are more accentuated. Correspondingly, a meticulous analysis of adjustment to select the most adequate probability distribution is recommended to estimate extreme events with return periods longer than 50 years.

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