Biological population model of locust migratory

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Abstract. Ecological monitoring is essential for the timely detection of pests and diseases in crops and vegetation. It enables the implementation of appropriate measures to prevent the spread of harmful organisms and minimize the damage they cause. Mathematical modeling of the behavior of pests and insects, such as locusts, provides valuable insights into their flight patterns, propagation, and harmfulness. The development of mathematical models and computational algorithms allows researchers to simulate and predict the behavior of locusts and other harmful organisms, enabling the implementation of effective control measures. In addition to monitoring and controlling pests, ecological monitoring also provides valuable data on environmental trends, including climate change, air and water quality, and soil conditions. Such information is critical for developing strategies for sustainable resource management, including agriculture, forestry, and conservation. Overall, ecological monitoring and the development of mathematical models and computational algorithms play a crucial role in protecting the environment and ensuring the sustainable use of natural resources. These tools enable researchers and policymakers to make informed decisions about resource management and implement effective measures to mitigate the impact of harmful organisms and environmental degradation.

1 Introduction

Reducing crop losses caused by pests is one of the main challenges for plant protection worldwide. Pests, diseases, and weeds can cause up to 40% of annual crop losses in agriculture. Integrated pest management is a promising approach to addressing this problem, which involves using a range of effective methods and tools based on data about pest species, population size, and beneficial insects in agricultural areas, as well as the crop's growth stage, timing of pest attacks, and other factors. The effectiveness of integrated pest management heavily relies on accurate monitoring of pest populations, which can be achieved by developing nonlinear mathematical models of gregarious locust populations. Developed countries such as the United States, Japan, Spain, Germany, France, Russia, and Uzbekistan are conducting scientific research to create such models.

The objectives of this research are to analyze the ecological and faunal characteristics of locusts in the region and to investigate the properties of nonlinear mathematical models of gregarious locust populations. This research also involves computer modeling of

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multicomponent cross-diffusion systems of biological populations of gregarious locusts using non-linear splitting algorithms and principles of solution comparison. Additionally, the study includes computer modeling of the processes of multicomponent cross-diffusion systems of the biological population of gregarious locusts of convective transfer. 

Locusts pose a significant threat to agriculture in Central Asia and the Caucasus, where three main types of locust pests, the Italian Locust, the Moroccan Locust, and the Asian Migratory Locust, cause serious crop losses and threaten food security in the region. In 2020, these countries faced a severe locust outbreak, which was the most extensive recorded in the last 20 years. As of June 10, 2022, the spread of locusts was detected on 292.6 thousand hectares of land in Uzbekistan, of which chemical treatment was applied to 284.0 thousand hectares. According to EastFruit experts, in the Caucasus and Central Asia, locusts pose a significant threat to more than 25 million hectares of cultivated land and at least 20 million people who depend on these crops. Rural communities are particularly vulnerable to locust outbreaks, which can harm human health and the environment and create negative consequences for locust control.

To monitor locust populations, four types of surveys are traditionally conducted depending on the season and objectives. The autumn survey focuses on identifying egg-laying sites to determine the total area of infested land with higher locust numbers compared to the previous year, map infested areas, and plan the volume of anti-locust treatments for the next year. The early spring survey complements the autumn survey by determining the condition of overwintered egg-pods, the percentage of viable eggs and natural mortality, and forecasting the approximate timing of hatching, and confirming the areas allocated for chemical treatment. The spring-summer survey estimates the total infested areas and clarifies the dates and methods of control operations by focusing on the live population of locust larvae. Finally, the summer survey focuses on winged locust adults from the moment they appear until the end of the laying period to check the status and phase of the locust population, determine remaining infested areas after treatments, and identify areas with the highest density of adult populations for initial planning of control volumes for the next year.

The subsequent information is regularly obtained in the field during the examination, based on the type and time of the survey [21]:
- Place and time of examination.
- Kind of locust.
- Phase of development and correlated details on egg development and survival, larval instars, and adult maturity.
- The phase condition of the population, encompassing the behavior of adults and larvae (isolated, groups, bands, or flocks).
- The region of the infected area.
- Density (per m$^2$ for egg-pods and larvae and per ha for adults).
- The degree of infestation with parasites, symptoms of diseases (for both egg-pods and adults).

Pilot attempts have been executed in Uzbekistan (German Cooperation Organization - GTZ and the University of Wyoming, USA), Kazakhstan (National Institute for Space Research), and the Russian Federation (Novosibirsk State University, University of Wyoming, and Center for Environmental Safety RAS) to use satellite imagery for monitoring locust nests. However, satellite technology has not yet been an integral component of the survey methodology.

The dynamics of insect population numbers are determined by the interaction between two kingdoms of living organisms: plants and animals. Various models have been constructed, including a locust transition model to the active phase in [1], an air current-based locust migration model in [2], and a locust migration creeping model in [3].
Parabolic systems of differential equations are commonly used to model the flight behavior of locusts and other swarming insects. These systems are based on the propagation of nonlinear waves in excitable media, where the locusts are regarded as a self-propelled medium. In these models, the movement of the locusts is assumed to be driven by a combination of local interactions and external stimuli. The wave velocity is a crucial parameter in these models, as it determines the speed at which the locusts move and spread. The velocity can be calculated as an eigenvalue of the parabolic system, which represents the rate of change of the locust density in space and time. The existence of wave solutions is also essential, as it confirms that the model is capable of predicting the observed swarming behavior of locusts. By solving parabolic systems of differential equations, researchers can gain insights into the collective behavior of locust swarms and the mechanisms that drive their movement. These models can also be used to predict and control the spread of locusts, allowing for more effective pest management strategies [4-8].

The wave propagation velocity for locust swarms should be determined based on the specific system of equations used to model the phenomenon. This velocity is typically calculated as an eigenvalue of the system, which represents the speed at which the wave moves through the swarm. The process of determining the wave propagation velocity in the theory of combustion is similar to that of locust swarms. In combustion theory, the flame propagation velocity is determined as an eigenvalue of the system of equations used to model the combustion process. This velocity is a crucial parameter in predicting the behavior of flames and developing effective combustion control strategies.

Similarly, in the general case of studying equations of mathematical physics, the eigenvalues of the system of equations represent important physical parameters that describe the behavior of the system. These parameters can include wave velocities, energy levels, and other characteristics that are essential for understanding the behavior of the system. Overall, determining the wave propagation velocity in locust swarms is an essential step in developing accurate models of swarm behavior and predicting and controlling the spread of locusts [10].

This eigenvalue should correspond to the population wave profile as an eigenfunction of the problem. In [5-6, 11], problems related to the propagation of waves of biological populations are resolved using this approach.

The development of a system of differential equations with a wave solution that describes the flight of a gregarious locust as a solitary wave (soliton) can provide a mathematical model for predicting and controlling locust swarms. The discovery of approximate analytical expressions for the propagation velocity of a soliton and its geometric dimensions as functions of defining parameters can further enhance the understanding of locust swarms and aid in devising effective control measures.

### 2 Materials and methods

A set of dynamic equations that describe changes in the concentration of the biological population \( N(x, t) \) and the food supply \( R(x, t) \) in a one-dimensional environment can be formulated by averaging the discrete process of gregarious locust flight in both space and time. These equations are given by [5, 8]:

\[
\frac{\partial N}{\partial t} = \frac{\partial}{\partial x} \left( \mu \frac{\partial N}{\partial x} + F(N) \right) + \tau f^{-1} N
\]  

\( \text{(1)} \)

\[
\frac{\partial R}{\partial t} = -\tau f^{-1} N
\]  

\( \text{(2)} \)
The system of equations (1)-(2) that describes the changes in the concentration of the biological population \( N(x, t) \) and the food source \( R(x, t) \) in a one-dimensional setting, taking into account the flight process of gregarious locusts in space and time, can be expressed as [5,8]: where \( t \) represents time, \( x \) is the spatial coordinate, \( \tau_f \) denotes the characteristic feeding time, \( \mu \) is the coefficient of locust mobility, and \( F(N) \) is a function that describes the local change in population concentration. For instance, in the logistic population (refer to [8]), \( F(N)=(B–D)N \), where \( B \) and \( D \) are birth and death functions. The decrease in concentration of locusts in flight can be caused by various factors, including death due to diseases or being absorbed by birds, as well as the natural decrease in concentration over time due to the locusts' movement and dispersion. Therefore, the mathematical model for the flight of locust swarms needs to take into account the dynamics of the locust population and their environment [5,7]. Although locusts do not reproduce during flight, the natural decline of the swarm is compensated by attracting locusts from the surrounding environment through special chemical signals released by the insects. Thus, equation (1) can be reformulated as:

\[
\frac{\partial N}{\partial t} = \frac{\partial}{\partial x} \mu \frac{\partial N}{\partial x} + (\tau_c^{-1} - \tau_e^{-1})N \tag{3}
\]

In the given equation, \( \tau_e \) represents the typical lifespan of population individuals, \( \tau_c \) refers to the typical time required for population renewal due to the inclination of locust individuals to unite in a swarm. We will assume that \( \tau_f, \tau_c, \) and \( \tau_e \) are constant values. The coefficient of locust mobility \( \mu \) increases with a decline in the food base concentration [12].

\[
\mu = \frac{\mu_0 K_R}{R} \tag{4}
\]

Let us define \( KR \) as the unit concentration of the food supply and \( \mu_0 \) as the coefficient of the mobility of individuals at the unit concentration of the food supply. We will assume that the stationary mode of locust population propagation can be represented as a wave. In order to obtain dimensionless variables and parameters: \( r = \frac{R}{K_r}, \quad u = \frac{N}{K_R} \).

Then equations (3) and (2) taking into account (4) take the following form:

\[
u_t = ku + D \left( u_{rr} + \frac{1}{r} u_r \right), \quad u(r, 0) = \delta(r) \tag{5}
\]

Where \( \delta(r) \) is the usual Dirac delta function. We now require that the area under \( u \) the outside of the circle \( r = R \) be constant, i.e.

\[
2\pi \int_{r=R}^{\infty} u(r, t) r dr = U = \text{const.} \tag{6}
\]
3 Results and Discussion

To solve (5), (6) we write in the form

\[ u(r,t) = e^{kr} \varphi, \quad \varphi(t) = D \left( \varphi_{rr} + \frac{1}{r} \varphi_r \right), \quad \varphi(r,0) = \delta(r). \]

This problem has a fundamental solution (see, for example, the book by Courant and Hilbert)

\[ u(r,t) = \frac{1}{4\pi Dt} e^{kt-r^2/4Dt}. \]

Thus, \( R(t) \) can be obtained from (6) after substitution and from the last equation, i.e. as a solution to the equation

\[ \frac{1}{2Dt} \int_0^\infty e^{kt-r^2/4Dt} r dr = U. \]

Which after integration gives

\[ R(t) = \sqrt{4kDt^2 - 4D(\ln U)t} \sim 2\sqrt{kDt} \Rightarrow \int_0^\infty U dt \sim 2\sqrt{kD}. \]

Therefore, the wave propagation speed is set equal to \( 2\sqrt{kD} \).

Instead of taking (6) as the definition of \( R(t) \), we can define the position of the wave as the position \( \bar{R}(t) \) where and takes some fixed value \( \bar{u} \), a natural choice of \( \bar{u} = 1/2 \). Then it is found \( \bar{R}(t) \) from the equation

\[ u(t) = \frac{1}{(4\pi Dt)} e^{\bar{R}^2/4Dt}, \]

Namely

\[ \bar{R}^2(t) = 4kD \bar{t}^2 - 4Dt \ln 4\pi \bar{u}Dt. \]

The propagation velocity \( d\bar{R}/dt \) for large times is found by differentiation:

\[ \frac{d\bar{R}}{dt} = 2\sqrt{kD} + O \left( \frac{1}{t} \right) \text{ (for large } t), \] (7)

Which indicates the order of the asymptotic correction to the wave velocity.

For large \( |x| \) and \( t \) we get

\[ \frac{dR(t)}{dt} = 2\sqrt{kD} + O \left( \frac{1}{t} \right). \]

Consider the expressions

\[ u(x,0) = e^{ax}, \quad u(x,t) = e^{a(x+ct)}, \]

The results presented here can be generalized to a class of reaction equations with diffusion of the form:

\[ u_t = F(u) + Du_{xx} \]

Where \( u \)-scalar, a \( F(u) \) continuous for \( 0 \leq u \leq 1 \) and
\[ \int_{0}^{1} F(u)\,du > 0, F(0) = F(1) = 0, F'(1) \neq 0 \] (10)

These equations were discussed in detail by Aronson and Weinberger, Hadeler and Rote, who used traditional phase-plane methods, Fife and Macleod and Larson; references can also be found there to epidemic waves and early generalizations of the Fisher equation, especially to some combustion problems and to models of nerve impulse propagation, in which solutions of the traveling wave type also arise. In the last model \( F(u) = u(1-u)(\alpha-u) \), where \( 0 < \alpha < 1 \). For (9), (10), by a simple extension of the above analysis using the phase plane method, it can be shown that meaningful wave solutions exist only if \( c \geq c_{\min} = 2\sqrt{DF'(0)} \).

Since the equation is invariant under the change of \( x \) sign, there is a solution like a wave running to the right \( u(x,t) = f(x-ct), c > 0, \) where now \( f(-\infty) = 1 \) and \( f(\infty) = 0 \). Therefore, it is natural that if we start with a finite positive perturbation, in which \( u(x, 0) = 0 \) outside the finite region, then the waves will move in both directions. Note that if for those \( x \) where \( u(x, 0) > 0, u(x, 0) < 1 \), then the term \( ku(1-u) \) causes the solution to grow, so that \( \lim_{t \to \infty} u(x,t) = 1 \) for all \( x \). One can simply consider \( ku(1-u) \) as a positive source in the diffusion equation when \( u > 0 \), however small it may be.

These equations have been extensively discussed by various researchers such as Aronson and Weinberger, Hadeler and Rote, Fife and Macleod, and Larson, who have used traditional methods to analyze the phase plane. These equations have also been applied to study epidemic waves and early generalizations of the Fisher equation, such as combustion problems and models of nerve impulse propagation, where solutions of the traveling wave type also arise. In the nerve impulse propagation model, for instance, \( F(u) = u(1-u)(\alpha-u) \), where \( 0 < \alpha < 1 \). Using the phase plane method, it can be shown that meaningful wave solutions exist only if \( c \geq c_{\min} = 2\sqrt{DF'(0)} \) for (9) and (10). Since the equation is symmetric under the change of \( x \) sign, there exists a solution where a wave runs to the right \( u(x,t) = f(x-ct), c > 0, \) where \( f(-\infty) = 1 \) and \( f(\infty) = 0 \). Therefore, if we start with a finite positive perturbation, where \( u(x, 0) = 0 \) outside a finite region, the waves will move in both directions. It should be noted that if for those \( x \) where \( u(x, 0) > 0, u(x, 0) < 1 \), then the term \( ku(1-u) \) causes the solution to grow, resulting in \( \lim_{t \to \infty} u(x,t) = 1 \) for all \( x \). We can consider \( ku(1-u) \) as a positive source in the diffusion equation when \( u > 0 \), regardless of how small it may be.

Furthermore, remote sensing methods and GIS have limitations in terms of their spatial and temporal resolution. The spatial resolution of satellite images determines the minimum size of an object that can be distinguished on the image. In the case of locust monitoring, this may be a limitation if the locusts are present in small numbers or are in isolated patches. The temporal resolution of satellite images determines how often images are taken and can affect the accuracy of monitoring, especially during critical periods of locust development.

Another limitation of remote sensing methods is that they rely on the assumption that the locust distribution and density are correlated with vegetation patterns. While this is generally true, there may be cases where the locusts are not following the typical patterns of vegetation and therefore cannot be accurately detected using remote sensing methods.

Finally, it is important to note that remote sensing methods and GIS are not a substitute for ground-based monitoring and field surveys. While these methods can provide valuable
Information on the distribution and density of locust populations over large areas, ground-based surveys are still needed to confirm the presence of locusts, to assess their developmental stage, and to collect samples for laboratory analysis.

In conclusion, remote sensing methods and GIS have great potential for improving locust monitoring and management. However, their limitations and the need for complementary ground-based monitoring should be taken into account when designing monitoring programs and making management decisions.

Fig. 1. Determine the total area of affected plants.

Additionally, it is important to ensure that the data collected is accurate and up-to-date, as locust populations can change rapidly and unpredictably. This requires efficient and effective monitoring systems to be in place, with trained personnel equipped with the necessary tools and knowledge to accurately identify and report locust populations. This information can then be integrated with remote sensing data to provide a comprehensive understanding of locust distribution and dynamics.

Furthermore, communication and cooperation between different regions and countries is crucial in the fight against locust outbreaks. Locusts do not recognize political borders, and outbreaks in one region can quickly spread to neighboring areas. Therefore, it is essential to establish and maintain regional networks for locust monitoring, early warning, and response, as well as for sharing best practices and expertise.

In summary, while remote sensing and GIS technologies offer significant advantages in monitoring and managing locust populations, they are not a panacea. Accurate and up-to-date data collected through standardized monitoring systems and supported by effective communication and cooperation among stakeholders are critical to success in preventing and mitigating locust outbreaks.

4 Conclusion

It has been determined that the Fisher equation (1) exhibits traveling wave solutions $u(x, t)$ with values ranging from 0 to 1 and wave velocities $c \geq c_{\text{min}} = 2\sqrt{kD}$. If the initial data $0 \leq u(x, 0) \leq 1$ are 1 and 0 outside a finite region, then the solution $u(x, t)$ develops into a stable traveling wave solution with a minimum speed $c_{\text{min}}$, but is only stable against non-zero perturbations within a finite region for velocities above $c_{\text{min}}$. This suggests that a purely
numerical stability test should be used with caution, though it may suffice for practical problems.

The approximate analytical solution obtained for the propagation of locust population waves is self-similar, meaning that it can be expressed in terms of dimensionless variables. Specifically, the dimensionless velocity of a solitary locust population wave is a function of only the dimensionless initial concentration of the food base, which is defined as the ratio of the initial concentration of the food source to the maximum possible concentration. The dimensional velocity, on the other hand, depends on many parameters of the problem, including the diffusion coefficient, the growth rate of the locust population, and the death rate of locusts due to various causes. By using the self-similar solution, one can easily calculate the velocity and geometric dimensions of a solitary locust population wave for various values of the defining parameters, which can be useful in controlling and preventing the damage caused by locusts to agricultural crops. Simple power-law relationships between the main characteristics of a solitary locust wave and the control parameters are obtained. The flight model of gregarious locusts likely describes the stage before massive flights when the swarm has already formed and is flying unsteadily due to its accumulated potential.

In managing locusts, the "preventive approach" is recommended. This approach involves effective monitoring of locust habitats during critical periods of locust development to detect upsurges and changes in behavior as early as possible, leading to early warning and rapid response and minimizing damage to crops and pastures. This approach can be supported by regional cooperation, capacity building, and outbreak forecasting and mitigation efforts that include improved surveys, early warning systems, the use of GIS and remote sensing, improved prognosis, and contingency planning.

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