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Bridging Mathematics and Cellulose Studies: Investigating Closed Subspaces Hilbert Base Construction

Sabarinsyah[1]*, *Ansarullah* Lawi[2], *Faradiba* Jabnabillah[3], and *Zelvin* Mutiara Lestari[4]

[1,3,4]Departement of Mathematics, Institut Teknologi Batam, Jl. Gajahmada Kompleks Vitka City, Batam 29425, Indonesia
[2]Departement of Industrial Engineering, Institut Teknologi Batam, Jl. Gajahmada Kompleks Vitka City, Batam 29425, Indonesia

Abstract. This research focuses on mathematically analyzing specific aspects of cellulose, particularly its molecular sequences. The goal is to employ Hilbert base construction to quantify and understand the structural and mechanical characteristics of cellulose at the molecular level. This is crucial for advancing our knowledge in plant biology and engineering cellulose-based materials. The study rigorously investigates mathematical properties, such as the equivalence between complete subspaces and closed subspaces, the interplay between closed subspaces and their orthogonal complements, and the existence of the closest vector within the Hilbert space to closed subspaces. The outcomes of this research can be adapted and extended to contribute to the construction and manipulation of Hilbert bases specifically for understanding and characterizing the polymer sequences in cellulose. Moreover, the application of these mathematical concepts extends to both structural and functional analysis of cellulose, encompassing mechanical behavior, chemical interactions, and functional attributes. This rigorous mathematical approach offers a more nuanced understanding of cellulose beyond its physical structure, paving the way for groundbreaking advancements in cellulose research and applications.

1 Introduction

Cellulose, a complex and integral component of plant cell walls, serves a crucial role in providing structural support and strength to various plant structures [1]. As we aim to deepen our understanding of plant biology and engineer customized cellulose-based materials, innovative mathematical approaches come to the forefront [2]. This research embarks on a journey to explore the potential utilization of Hilbert base construction within the context of cellulose studies. The mathematical elegance of Hilbert spaces offers a gateway to delve into the intricate spatial attributes and mechanical behavior of cellulose.

In this study, the concept of sequence of numbers is assumed to be similar to the concept of cellulose molecular chains, because cellulose is consist of long molecular chains, of which the links are β-glucose residues bound together, in the chain, by primary valences [3]. And concept of closed subspace is assumed to be similar with concept of microfibrils, because cellulose fibrils are synthesized and grow extracellularly, they push up against neighboring cells. Since the neighboring cell cannot move easily the Rosette complex is instead pushed around the cell through the fluid phospholipid membrane. Eventually this results in the cell becoming wrapped in a microfibril layer. This layer becomes the cell wall. Then concept separable Hilbert space is assumed to be similar with concept cellulose fiber because cellulose fibers are fibers made with ethers or esters of cellulose. the fibers may also contain hemicellulose and lignin, with different percentages of these components altering the mechanical properties of the fibers.

In accordance with the three assumptions above, the topic of research about the construction of the Hilbert base of a closed subspace in separable Hilbert space can be applied. This topic was chosen because algebraically there is a relationship between the concept of orthogonal complement subspace with the structure of related subspaces [4]. The exploration encompasses several significant properties of closed subspaces. Specifically, a subspace is said to be complete if and only if every sequnce in that subspace converges to one element in the same subspace [5]. Furthermore, a closed subspace is a subspace that has orthogonal complement properties from its orthogonal complement is a related subspace. In addition, in the Hilbert space applies the property of direct summation between any closed subspace and its orthogonal subspace [5]. The adaptability of these closed subspace properties to construct a Hilbert basis underscores their potential for analyzing cellulose behavior in innovative ways.

2 Materials and Methods

In this section, we outline the key mathematical concepts and properties related to separable Hilbert spaces and closed subspaces that are central to our

* Corresponding author: sabarinsyah@iteba.ac.id

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research on the potential application of Hilbert base construction in cellulose studies.

Definition 1 Suppose V is the inner product space over the field $\mathcal{F}$. Any subspace S of V is said to be complete if every Cauchy sequence in S converges to an element in S [6].

Theorem 2 Suppose H is a Hilbert space. Then every subspace S of H is complete if and only if S is a closed subspace [6].

Theorem 3 Suppose H is a Hilbert space and S is any closed subspace of H. For each $\mathbf{x}$ in H there is the only one $\hat{\mathbf{s}}$ in S such that

$$(\mathbf{x} - \hat{\mathbf{s}}) \text{ is contained in } S^{\perp}$$

where $S^{\perp}$ is the orthogonal complement of the S [5].

Corollary 4 [5] Suppose H is a Hilbert space and S is any closed subspace of H. Then

i. $H = S \oplus S^{\perp}$

ii. $(S^{\perp})^{\perp} = S$.

Definition 5 Let V be the space of the inner product over the field $\mathcal{F}$. The subset S of V is said to be separable if S contains a countable dense subset [5].

Theorem 6 [6] Let V be the space of the inner product over the field $\mathcal{F}$. The orthonormal subset M of V is said to be maximal if

$$\mathbf{x} \in V, \quad \mathbf{x} \perp M \Longrightarrow \mathbf{x} = \mathbf{0}$$

Definition 7 Suppose H is the Hilbert space. The subset $M(\neq \emptyset)$ of H is said to be the Hilbert basis of H if M is the maximal orthonormal set [6].

3 Results

3.1 Temporary Result

Let H be a separable Hilbert space over a field $\mathcal{F}$ and W any closed subspace of H. Suppose given

$$\mathcal{B} = (\mathbf{x_1}, \mathbf{x_2}, \cdots)$$

is a set of Hilbert base of H. Define

$$U_i = W \cap V_i \; ; \; i \in \mathbb{N} \cup \{0\}$$

where

$$V_i = C\ell[span(\mathbf{x_{i+1}}, \mathbf{x_{i+2}}, \cdots)] \; ; \; i \in \mathbb{N} \text{ and } V_0 = H$$

Then the following statements hold

1. For some $i \in \mathbb{N}$:

$$U_i = U_{i+1} \quad \text{or} \quad \begin{cases} U_i = span(\tilde{\mathbf{x}}_\mathbf{i}) \oplus U_{i+1} \\ \text{where } \tilde{\mathbf{x}}_\mathbf{i} \text{ in } U_i, \\ \|\tilde{\mathbf{x}}_\mathbf{i}\| = 1 \; ; \; \tilde{\mathbf{x}}_\mathbf{i} \perp U_{i+1} \end{cases}$$

2. The set

$$X = (\tilde{\mathbf{x}}_\mathbf{i} | i \in I) \quad \text{where} \quad I = (i \in \mathbb{N} | U_i \neq U_{i+1})$$

is basis Hilbert of W [7].

The findings presented above lead to the conclusion that, through the utilization of the Hilbert basis of any separable Hilbert space, one can construct the Hilbert basis of any closed subspace. In essence, to analyze the properties of a separable Hilbert space, it suffices to investigate the properties of any closed subspace.

3.2 Implications and Significance

The results of this study will have implications for the use of Hilbert base construction in cellulose studies. Leveraging the characteristics of closed subspaces within separable Hilbert spaces, we can proficiently investigate cellulose behavior. The capacity to create Hilbert bases for these closed subspaces presents fresh opportunities for gaining insights into the structural and functional dimensions of cellulose, with the potential to drive progress in the development and application of cellulose-based materials. Figure 1 and Figure 2 illustrate the construction of the Hilbert base, representing the construction process of the Hilbert base from the molecular chain of cellulose Each chain of cellulose molecules is described as a sequence of numbers

$$(a_n | n \in \mathbb{N})$$

where a_n representing a molecular unit within the chain. By representing the cellulose molecular chain as a sequence of numbers in the Hilbert base studied, we can describe a variety of possible cellulose configurations.

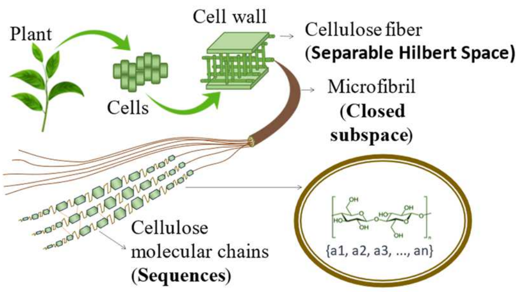

Fig. 1. Conceptual Representation of Microfibril within Hilbert Base Contruction.

Detailed image of the Conceptual Representation of Microfibrils within Hilbert Base Contruction is shown in the following figure

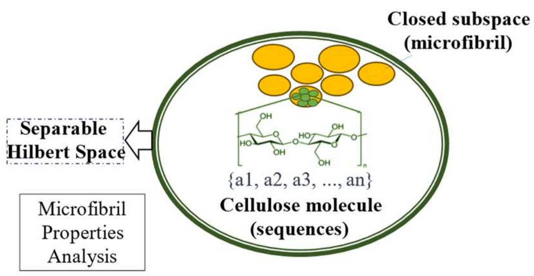

Fig. 2. Detailed Image of the Conceptual Representation of Microfibril within Hilbert Base Contruction.

This approach offers a systematic organization of cellulose chains, allowing for a comprehensive examination of their properties. In this constructed Hilbert base, we represent a closed subspace as a microfibril corresponding to a specific structural component of cellulose. This focus on microfibrils as closed subspaces offers a targeted approach to delve deeper into the mechanical, chemical, and structural characteristics of cellulose at the microfibril level. While cellulose is traditionally studied at the macroscopic level, understanding its behavior at the molecular and microfibril scales is pivotal for applications in biofuel production, materials science, and more [8]. By leveraging mathematical constructs like the Hilbert base, we bridge the gap between theoretical abstraction and practical cellulose research. The insights gained from analyzing closed subspaces, such as microfibrils, provide a foundation for making informed inferences about the properties of the entire cellulose structure.

4 Conclusion

In conclusion, the application of Hilbert Basis construction opens a promising avenue for unraveling cellulose mysteries. By employing mathematical constructs, we can potentially discern the intricate dance between the original polymer sequences predating cellulose and the subsequent modifications within cellulosic sequences. This mathematical framework provides a unique lens to explore the conservation of polymeric structures and assess the extent of modifications, injecting a quantitative dimension into the evolution of cellulose. The transformative power of Hilbert construction becomes evident as it emerges from the abstract realm to shed light on tangible properties.

Moreover, the introduced mathematical concepts not only empower us to investigate but also to quantify cellulose's structural, chemical, and mechanical nuances. Taking a focused standpoint, let's delve into the mechanical realm, specifically stiffness. The Hilbert construction space, a meticulous mathematical model, serves as a lens to scrutinize the fluctuations in mechanical strength. Through this lens, we can discern subtle variations, whether in the form of a decrease or an increase in stiffness. The mathematical precision embedded in Hilbert Basis construction allows us to translate abstract concepts into tangible shifts in cellulose properties, forging a direct link between theoretical understanding and practical implications.

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