

Developing Rigorous Mathematical Thinking Approach-Based Learning Materials for Sequences and Series Concept at High School Level in Tanjungpinang

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Abstract. Mathematics encourages logical reasoning and critical thinking. Rigorous thinking helps in analysing problems to decision-making and ensures that solutions are accurate and reliable. Prior research indicated students' lack of those abilities when learning the concept of sequences and series. Therefore, this research aims to develop learning materials for the concept of sequences and series based on rigorous mathematical thinking (RMT) approach. The learning materials developed with the ADDIE model include mobile learning, student worksheets, and test instruments, which provide clear explanations, examples, problem-solving activities with RMT approach, and assessments enabling teachers to evaluate students' understanding of mathematics concepts of the sequences and series. The learning materials were validated using a questionnaire by mathematics experts. The quantitative and qualitative data results show that the mobile learning, student worksheets, and test instruments are valid. Mobile learning that was developed and tested valid entails that the study can add to a new learning strategy that proposes technology and digital tools to engage students in the learning process and promote RMT.

1 Introduction

The Indonesian Ministry of Education, Culture, Research, and Technology (Kemendikbud) emphasizes preparing students through the education system to face the real world [1]. Therefore, the national curriculum provides students with academic or subject knowledge, character development, life skills, and values to face the 21st century [2][3][4].

Despite education as a process of developing 21st-century competencies : creativity, critical thinking, collaboration, and communication (4C) [5], when in the process of learning mathematics subject, the skills are even required to gain an understanding of mathematics concepts itself. Mathematics is a problem-solving activity [6][7][8]. Students' errors in understanding the problems and applying formulas can lead to wrong answers. Therefore,

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mathematics is a precise and rigorous subject. A deep understanding of mathematics and skills developed during the process prepare students to solve problems and make decisions in the real world.

When dealing with mathematics problems, with its character of problem-solving, rigorous mathematical thinking is crucial to ensure that students' decisions and solutions are accurate and reliable. This situation in mathematics class may have significant real-world implications when students face real-world problems. Additionally, rapid decision-making and problem-solving are now in the global even very important, making a rigorous mathematical approach even more critical to prepare students for future challenges.

Rigorous approach is crucial in teaching and learning mathematics to help students grasp the concept and develop strong foundational skills [9][10]. A rigorous approach helps present mathematical concepts clearly and logically [11]. Therefore, students can develop a deeper understanding of mathematical ideas and be more confident in mathematics.

Hence, investment in thoughts on how mathematics in high school is taught is considerably needed. One aspect teachers must prepare in teaching is learning materials. Nevertheless, references mention that mathematics teachers face several challenges when developing learning materials for their students [12][13][14]. Some challenges include : How should a concept be given and taught ? How is the learning design that can be used to deliver concepts simultaneously develop students' skills and give students an interactive learning experience : to facilitate teacher interaction with students, fellow students, and students with the material ? And hence, with these questions, what media is effective ? Can it be integrated with technology ? And how ? Last, how to assess students' understanding and progress ? A scientific approach, such as rigorous mathematical approach, can support teachers in designing instructions.

While learning materials are an important factor in improving the quality of learning and abilities of students, which many teachers still struggle in response to, another challenge is students' difficulties in learning the mathematics concept. Some research reports on students' weak understanding of the concepts of sequence and series [15] [16]. Specifically in Tanjungpinang in Riau Archipelago Province in Indonesia, from the interview with a high school (SMA) mathematics teacher, it is known that students face difficulties in defining mathematical ideas and concepts and using formulas in solving problems related to sequences and series, leading to difficulties in understanding the sequence and series topic. Additionally, the teacher informed that the learning materials used when teaching were mathematics textbooks and learning videos. The teacher usually took questions from the books and changed the number to assess students' achievement. The teacher is not yet concerned about the process of constructing the concept encouraging 4C skills with rigorous thinking.

Various research has been focused on studies related to the concept of sequence and series. Some analysed students' conceptual understanding [17][18], students' problem-solving skills [19], and focused on obstacles [20]. However, there is still a lack of studies about learning materials specifically developed to address the difficulties faced by students and mathematics teacher in learning sequences and series in Tanjungpinang. Additionally, developing learning materials based on rigorous approach that specifically targets the concept of sequences and series is still not yet done.

Rigorous Mathematical Thinking (RMT) approach was introduced by Kinard [9] and has been used in some research [11][21]. They show students' better mathematics understanding and achievement of mathematical critical thinking and creative thinking [22–24]. With positive impact, however, these research have not provided learning materials to help teachers implement RMT approach in the learning process, as well as to support students' understanding and skills, which are complete, from media to test instrument, and utilizing technology-based materials, and to further develop our previous research [22][23].

Overcoming the challenges, in response to teacher’s and student's needs in the field, and evolving educational policy, in this research, we aim to develop valid learning materials that give theoretical and practical examples of integrating RMT approach in learning mathematics, especially the concept of the sequences and series of SMA class. This paper describes the process of developing valid learning materials for the concept sequences and series based on RMT approach.

2 Method

This research aims to develop a valid set of learning materials of sequence and series concept of XI high school with RMT approach. The teaching materials are mobile learning to deliver the concept, electronic student worksheets (E-LKPD) to provide interactive activities to construct the concept, and test instruments to assess students' progress; all are addressed with specific preference to implement RMT approach. Therefore, ADDIE model Analysis, Design, Development, and Implementation is used to meet the research aim. This paper describes the process of analysis, design, and development. When developing, we refer to the Curriculum 2013 revised 2017 edition as this curriculum is implemented where this research was conducted, in the academic year 2022/2023, in Class XI SMAN 4 Tanjungpinang.

The data to measure the validity of the learning materials are obtained from the validity instrument, which consists of statements in aspects: material delivery, media, and language. The validity questionnaires use Likert scale of 1 to 5 (as seen in Table 1) to assess the quality of the learning materials [25]. In addition, the questionnaires are provided with additional comments, suggestions, or concerns regarding the learning materials. The quantitative and qualitative data collected are analysed descriptively.

Table 1. Scoring of Each Statement.

Scale	Description
1	Strongly Disagree
2	Disagree
3	Neutral
4	Agree
5	Strongly Agree

To analyse the Likert scale results of the validity questionnaires, the validity score is calculated with the formula (1)

$$validity\ percentage\ (\%) = \frac{total\ score}{maximum\ score} \times 100\% \tag{1}$$

The result is interpreted as seen in Table 2.

Table 2. Valid interval category.

Interval Coefficient	Interpretation
0% - 19,99%	Not Valid at All

20% - 39,99%	Somewhat Valid
40% - 59,99%	Not Very Valid
60% - 79,99%	Valid
80% - 100%	Very Valid

3 Results and Discussion

The ADDIE model to develop the teaching materials:

3.1 Analysis

In this phase, the objectives of the teaching materials are defined by analysing Curriculum 2013 revised edition 2017 in Class XI of Senior High School for Sequence and Series Concept. The outline to learn this basic concept of arithmetic is provided in Table 3 Basic Competence 3.6 is a process to investigate the sequential arrangement of numbers to find the general formula or a specific rule of the arrangement, which is defined as a sequence and to find the sum of the number elements in the sequence which is defined as the series. Besides, there problem-solving skills are to develop to apply the sequence and series concepts to solve real-world problems.

Table 3. Basic competence and Competency achievement indicator.

Basic competence	Indicator
3.6 Generalize number and sum patterns in Arithmetic and Geometry sequences	3.6.1 Determine the number terms in an arithmetic sequence.
	3.6.2 Find the formula for the sum of an arithmetic series.
	3.6.3 Determine the number terms in a geometric sequence.
	3.6.4 Find the formula for the sum of a geometric series.
	3.6.5 Find the formula for the sum of an infinite geometric series.
4.6 Use arithmetic or geometric sequence and series to present and solve contextual problems	4.6.1 Solve contextual problems related to arithmetic sequences.
	4.6.2 Solve contextual problems related to arithmetic series.
	4.6.3 Solve contextual problems related to geometric sequences.
	4.6.4 Solve contextual problems related to geometric series.

4.6.5 Resolve related contextual problems

Hence, Table 3, shows that students need to understand concepts first which means they build a foundation in the concepts and then apply them to contextual problems. Learning settings can also be designed involving problem-solving skills when students construct the concepts. With this learning situation, students are guided with RMT approach to construct conceptual understanding presented in Table 3 and make rigorous arguments when solving problems.

As stated in [11], RMT is a cognitive function as well as a learning approach. RMT is based on two theories, Vygotsky's sociocultural theory and the zone of proximal development (ZPD), and Feuerstein's Mediated Learning Experience (MLE) and cognitive function theory. Vygotsky's socio-cultural theory emphasized in the theory of RMT is the concept of psychological tools, which include signs and symbols, graphic/symbolic organizers, formulas and equations, and mathematical language. ZPD concept in developing RMT is analysis of those emerging psychological functions that provide the prerequisites of rigorous mathematical reasoning. Meanwhile, the Mediated Learning Experience (MLE) theory is applied to mediated learning using cognitive tasks designed to develop deductive thinking from general to more concrete models [9,10]. Therefore, RMT approach highlights the role of cognitive tools and social interaction in learning.

The cognitive function of RMT is characterized by three levels, namely qualitative thinking cognitive : at this level, students use the abilities they already understand, quantitative cognitive : students accurately solve mathematical problems, and abstract relational cognitive : students reactivate previous mathematical knowledge to define problems and then carry out problem-solving on the problems given [9].

Thus, students construct the concepts of sequence and series based on RMT cognitive level indicators. The general cognitive functions for qualitative thinking level indicator consists of labeling-visualizing, comparing, and searching systematically to gather clear and complete information, using more than one source of information, encoding-decoding. The cognitive function for quantitative thinking with precision level indicator consists of conserving constancy, quantifying space and spatial relationships, quantifying time and temporal relationships, analyzing-integrating, generalizing, and being precise. The cognitive functions for generalized, logical abstract relational thinking in the mathematics culture level indicators consist of activating prior mathematically related knowledge, providing and articulating mathematical logical evidence, defining the problem, inferential-hypothetical thinking, projecting and restructuring relationships, forming proportional quantitative relationships, forming a functional relationship, forming a unit functional relationship, mathematical inductive-deductive thinking, mathematical analogical thinking, mathematical syllogistic thinking, mathematical transitive relational thinking, elaborating mathematical activity through cognitive categories [9].

In alignment with the learning objectives of sequence and series concepts in the curriculum, learning materials are developed to build interactions (see Figure 1) to mediate students in constructing the cognitive processes using the three-phase of RMT : Cognitive development, Content as process of development, and Cognitive conceptual construction practice. Cognitive development involves mediating students to model the tasks as a general psychological tool, and then use the tool to construct higher-order cognitive processes. In content as process development, students are mediated to build basic concepts, formulate mathematical patterns and relationships, and appropriate mathematically specific psychological tools based on their relationships. In phase 3, cognitive conceptual construction practice, students are mediated to use each mathematically specific psychological tool to construct mathematical conceptual understanding [9].

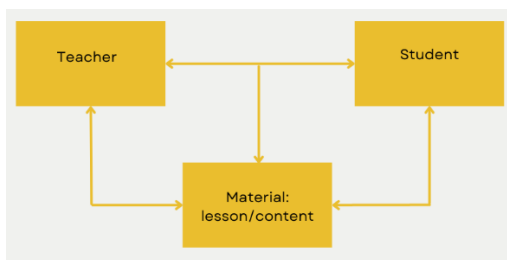


Fig. 1. Rigorous engagement for RMT

3.2 Design


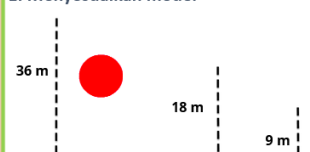
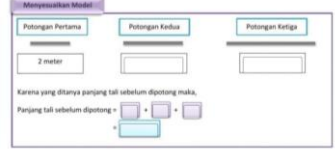
In this phase, we create an outline of the mathematics content in our learning materials, the design structure and format, and a storyboard of the flow of the materials. The learning materials include technology-based and interactive learning, complemented with the RMT approach. While others address developing technology-based learning material [26] without considering a specific approach and developing learning material with an approach [27][11] without integrating technology, we combine the benefits of technology with an instructional approach. Both aspects provide students with an interactive and engaging learning process and also ensure that the process aligns with the learning objectives.


The technology used in developing learning material is mobile learning and electronic worksheets (E-LKPD). Mobile learning is a way to mediate mathematical content to students and provides unlimited learning opportunities in terms of space and time. Research on mobile learning to learn mathematics has shown positive effects on student learning outcomes, particularly for students in medium and low categories [28] [29]. Electronic worksheets mediate problem-solving activities for students with interactive learning experiences. The use of electronic worksheets to learn mathematics has also been shown to have positive outcomes [30] [31]. As stated in [9], rigorous engagement for RMT provides bidirectional interactions between teacher and student, teacher to material, and student to material (see Figure 1).

The sections of mobile learning are the homepage, main menu, and logout page. Each section is designed in Microsoft PowerPoint first and then mobile learning application is developed using Articulate Storyline 3 and Website Apk Builder Pro. The homepage provides cover, user identity, and menu selection. The main menu provides lesson pages, including an overview of the available features and content, basic competence and competency achievement indicators, mathematics history, materials explanations with interactive content, and exercises to review students' conceptual understanding. The Logout page provides developer information. In the material explanation part, students are mediated to understand the concept of sequence and series with the help of the phases [9] of the RMT concept of understanding; also, students receive a response from each column filled (see Table 4) (we give an example of the conceptual development of Geometric sequence).

Furthermore, E-LKPD uses the Liveworksheets application to offer engaging activities for learning. The sections are the homepage to access cover, student identity, and some icons; the main menu includes basic competence and competency achievement indicators, learning objectives, video and instruction activities to learn materials with RMT phases [9] ; and a logout page.

Table 4. The RMT process of conceptual development.

The RMT phase	Lesson Page at Mobile Learning	Lesson Page at E-LKPD								
Stimulating	<p style="text-align: center;">Geometri</p> <p>A. Mengkonsepkan Barisan Geometri</p> <p><i>Stimulation</i></p> <p>Perhatikan permasalahan berikut!</p> <p>Ade menjatuhkan bola dari ketinggian 36 meter dari permukaan tanah, kemudian bola tersebut memantul dengan setinggi 18 meter, bola tersebut jatuh kembali dan memantul setinggi 9 meter. Setiap bola memantul ketinggiannya selalu berkurang dengan pola yang sama. Bantu Ade untuk megkira-kira ketinggian bola setelah memantul sebanyak 5 kali.</p> <p><i>From the picture, at geometric sequence topic, students are given task to find the height of the fifth time of a bouncing ball. The ball is dropped from the height of 36m and touched the ground and bounced 18 m, and so on.</i></p>	 <p><i>It can be seen from the picture, at E-LKPD of finite geometric series topic, students are given a task and learning video. The task is to determine the initial length of rope cut into 3 sections of increasing length 1,5 times and known that the first cut is 2m.</i></p>								
Phase 1: Cognitive Development	<p style="text-align: center;">Level Berpikir Kualitatif</p> <p><i>Cognitive Development</i></p> <p>1. Menyesuaikan model</p>  <p>Dari ilustrasi di atas, mari kita kumpulkan informasi yang didapatkan</p> <p>*Perkirakan jawaban dan isilah pada kotak biru di dalam tabel</p> <table border="1" data-bbox="399 1256 695 1370"> <thead> <tr> <th>Bola saat</th> <th>awal (U₁)</th> <th>Pantulan ke 1 (U₂)</th> <th>Pantulan ke 2 (U₃)</th> </tr> </thead> <tbody> <tr> <td>Ketinggian Bola</td> <td>36</td> <td style="background-color: #00aaff; width: 30px; height: 30px;"></td> <td style="background-color: #00aaff; width: 30px; height: 30px;"></td> </tr> </tbody> </table> <p style="text-align: center;"> Back Cek Next </p> <p><i>Students are mediated to model the task (see the illustration of the bouncing ball)</i></p>	Bola saat	awal (U ₁)	Pantulan ke 1 (U ₂)	Pantulan ke 2 (U ₃)	Ketinggian Bola	36			<p>Berdasarkan permasalahan diatas, cobaah temukan jarahannya menggunakan visualisasi / kalkulasi seperti langkah dibawah.</p> <p><i>Menyesuaikan model</i></p>  <p>Karena yang ditanya panjang tali sebelum dipotong maka, Panjang tali sebelum dipotong = $2 + 2 + 2 = 6$</p> <p>Dari persoalan diatas jumlah potongan tali masih sederhana. Bagaimana jika jumlah potongan tali lebih banyak dari itu, apakah efektif menggunakan langkah seperti diatas?</p> <p>Oleh karena itu mari kita temukan bentuk umum serta rumus deret geometri, agar lebih mudah dalam menyelesaikan persoalan terkait deret geometri.</p> <p><i>Students are mediated to model the sum of length of rope (see the illustration above)</i></p>
Bola saat	awal (U ₁)	Pantulan ke 1 (U ₂)	Pantulan ke 2 (U ₃)							
Ketinggian Bola	36									



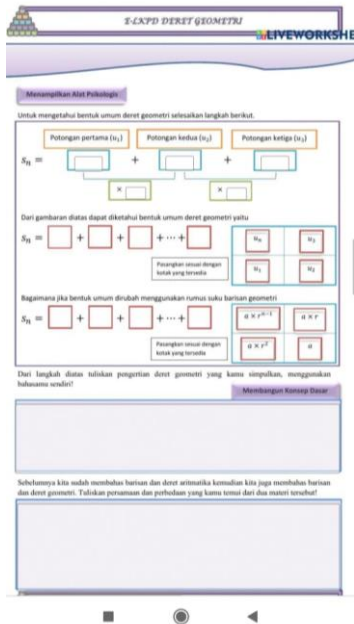
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 2. Menampilkan Alat Psikologi

Dari data yang telah kita peroleh, mari kita identifikasi pola dari barisan aritmatika dengan melengkapi gambar berikut ini!

U_1	U_2	U_3	U_4	U_5
36	18	9		

$\frac{1}{K_2} \times \frac{1}{K_2} \times \frac{1}{K_2} \times \frac{1}{K_2}$

as psychological tools (identifying each drop as the series U_1, U_2, \dots, U_5) and use the tools to construct higher order cognitive process (find the ratio and the U_5)



E-CXPD DERET GEOMETRI
 L1VEWORKSHO

Mengidentifikasi Pola

Untuk mengetahui bentuk umum deret geometri selanjutnya lakukan berikut.

Potongan pertama (u_1) Potongan kedua (u_2) Potongan ketiga (u_3)

$S_{10} = \square + \square + \square + \dots + \square$

Dari gambaran data dapat diketahui bentuk umum deret geometri yaitu:

$S_{10} = \square + \square + \square + \dots + \square$

Bagaimana jika bentuk umum diubah menggunakan rumus suku barisan geometri?

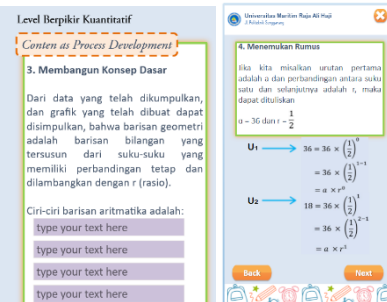
$S_{10} = \square + \square + \dots + \square$

Dari langkah diatas salahkan pengantar deret geometri yang kamu simpulkan, menggunakan bahasaamu sendiri!

Sebelumnya kita sudah membahas barisan dan deret aritmatika kemudian kita juga membahas barisan dan deret geometri. Tolaklah persamaan dan perbedaan yang kamu temui dari dua materi tersebut!

as psychological tools (see picture below, students identify the sequence of the form $U_1+U_2+U_3+\dots+U_n$ and $a+ar+ar^2+\dots, ar^{n-1}+\dots$) and use the tools to construct higher order cognitive process

Phase 2:
 Content as
 Process of
 Development



Level Berpikir Kuantitatif
 Content as Process Development

3. Membangun Konsep Dasar

Dari data yang telah dikumpulkan, dan grafik yang telah dibuat dapat disimpulkan, bahwa barisan geometri adalah barisan bilangan yang tersusun dari suku-suku yang memiliki perbandingan tetap dan dilambangkan dengan r (rasio).

Ciri-ciri barisan aritmatika adalah:

type your text here

type your text here

type your text here

type your text here

4. Menemukan Rumus

Eka kita misalkan urutan pertama adalah a dan perbandingan antara suku satu dan selanjutnya adalah r , maka dapat dituliskan:

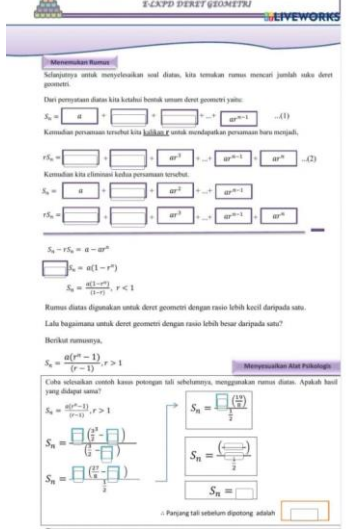
$a = 36$ dan $r = \frac{1}{2}$

$U_1 \rightarrow 36 = 36 \times \left(\frac{1}{2}\right)^{1-1}$
 $= 36 \times \left(\frac{1}{2}\right)^{1-1}$
 $= a \times r^{n-1}$

$U_2 \rightarrow 18 = 36 \times \left(\frac{1}{2}\right)^{2-1}$
 $= 36 \times \left(\frac{1}{2}\right)^{2-1}$
 $= a \times r^n$

$U_n \rightarrow U_n = a \times r^{n-1}$

Students are mediated to build basic concept (see picture, students are asked to identify characteristics of geometric sequence), discover (to consider the geometric sequence of the task) and specify psychological tools formula (build pattern and relationship and find



E-CXPD DERET GEOMETRI
 L1VEWORKSHO

Mengembangkan Rumus

Sebelumnya untuk menyelesaikan soal diatas, kita gunakan rumus umum jumlah suku deret geometri.

Dari pernyataan diatas kita ketahui bentuk umum deret geometri yaitu:

$S_n = a + \square + \square + \dots + \square$

Kemudian persamaan tersebut kita kalikan untuk mendapatkan persamaan baru menjadi,

$rS_n = \square + \square + \square + \dots + \square$

Kemudian kita eliminasi kedua persamaan tersebut.

$S_n = a + \square + \square + \dots + \square$

$rS_n = \square + \square + \square + \dots + \square$

$S_n - rS_n = a - a \times r^n$

$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

Rumus diatas digunakan untuk deret geometri dengan rasio lebih kecil daripada satu.

Lalu bagaimana untuk deret geometri dengan rasio lebih besar daripada satu?

Berikut rumusnya,

$S_n = \frac{a(r^n - 1)}{r - 1}$

Coba selesaikan contoh kasus potongan tali sebelumnya, menggunakan rumus diatas. Apakah hasil yang didapat sama?


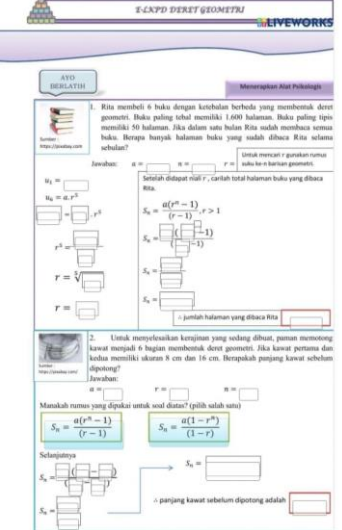
$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

Paragraf tali sebelum dipotong adalah

Students are mediated to build basic concept (see picture, students are asked to identify the sum of terms that have common ratio between adjacent two of them with a as the

	<p><i>that a is the first term, r is the ratio, and the formula of nth term)</i></p>	<p><i>first term), discover and formulate the patterns (from the pattern, students find the formula of sum of n term of a finite geometric sequence)</i></p>
<p>Phase 3 : Cognitive Conceptual Construction Practice</p>	 <p><i>Students are mediated to practice the use of psychological tool to construct understanding of geometric sequence concept.</i></p>	 <p><i>Students are mediated to practice the use of psychological tools to construct understanding of geometric series concept</i></p>

Mathematical concepts built into the learning materials of this research are arithmetic sequence and series, geometric sequence, and finite and infinite geometric series. In Table 4, we present an example of mobile learning about geometric sequences and an example of E-LKPD about geometric series. In phase 1, RMT cognitive level that students build is qualitative thinking, where students are situated to appropriate the models and perform tasks through labelling-visualizing, comparing, and searching for clear and complete information of the task. In phase 2, students develop quantitative thinking with precision, where they are mediated to build concepts, find formulas, and appropriate the tools by conserving constancy, quantifying relationships, generalizing, and being precise. In phase 3, students are developing the cognitive for abstract relational thinking, where they activate prior concepts, organize tools, and perform mathematical inductive-deductive thinking to solve problems.

The test instrument is designed to consist of a content outline, question items in alignment with competency achievement indicators with RMT level, alternative answers, and scoring guidelines, each consisting of two packages of ten questions. However, the test instrument is developed only at RMT qualitative level [9] (see Figure 2 for a task about the sum of infinite geometric series).

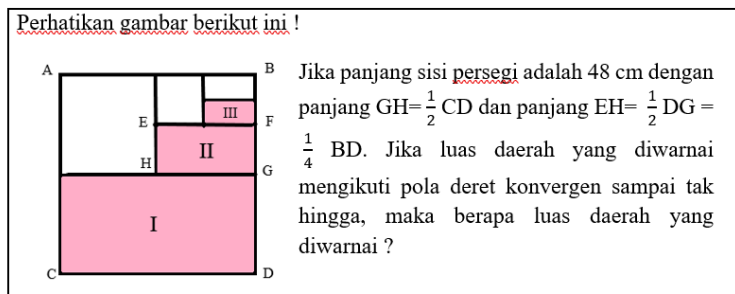


Fig. 2. Qualitative level thinking question of geometric series

This type of question mediates students to answer by labelling-visualizing (producing an internalized construction of an object), comparing (looking for similarities and differences between objects), and searching systematically to gather clear and complete information (looking in an organized way to collect clear and complete information), using more than one source of information (working with 2 or more concepts), and encoding-decoding (putting meaning into a symbol/sign). We identify students can do labelling-visualizing when they can break the picture and specify the information of the length side of each small picture (see the alternative answer in Figure 3); they can do searching systematically to gather clear and complete information when they figure and work out the area of objects of which sides are known (Figure 3); can do comparing when they write pattern of infinite geometric sequence of the task; and working with 2 concepts when they can use the information from the sequence which is the first term as a and r as the common ration and use them for the formula of the sum of infinite geometric series; can do encoding-decoding if they can look back the answer of S to the task context.

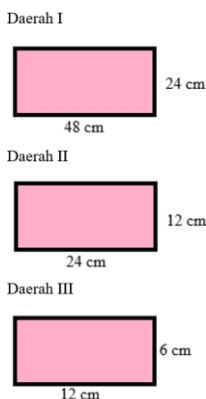


Fig. 3. Labeling-visualizing at qualitative level

Additionally, to identify any issues for improvement of the teaching materials, we gather expert feedback regarding the concept delivery, the media, and the language with validation questionnaires. The validity aspects of concept delivery include alignment with basic competence and indicators, clarity of learning objectives, correctness of the substance of the learning materials, conformity of the learning materials with RMT approach (cognitive development, content as a process development, and cognitive conceptual construction practice), and presentation of materials. Especially for the electronic worksheet, we add question items about the feasibility of components of the electronic worksheet [32]. The validity aspects of media include question items about appearance, the font type, the menus,

and the application. The validity aspects of languages include readability, information clarity, the correctness of Indonesian language rules, and terms used. For the test instrument, we measure the validity of content, construct, and language aspects.

3.3 Development

In this phase, we create the actual learning materials based on the design.

3.3.1 Mobile Learning

Considering learning objectives in curriculum standards in Sequence and Series concepts, we develop the content of concept delivery integrating RMT approach. The RMT approach helps to develop students' experience for students when learning the concept.

Additionally, to ensure the quality of mobile learning, we seek feedback from 2 experts to refine the content and instructions: a lecturer of a mathematics education study program and a mathematics teacher at SMAN 4 in Tanjungpinang. The validation process involves the material, the language, and the media. The 13 questionnaire items of material validation results from the two experts are analysed descriptively. The obtained average score is 89,23, in very valid criteria. Also, the expert validation results give some recommendations to add a discussion menu to the exercise problems menu and suggestions for word clarity in some instructions in mobile learning.

The 15 questionnaire items of media validation results from the experts are also analysed descriptively with an average score 89,33, in very valid criteria. The expert validation results on media give comments that the presentation of content, menus, and displays in the application is good and can be operated smoothly. Also, the suggestion is to revise the theme used in the main menu page to be more suitable with the sequence and series concept.

On average, the language validation score is 92,0 in very valid criteria. An expert from the lecturer in the Indonesian Language and Literature Education study program and a mathematics teacher give suggestions: 1) to be more consistent in presenting mathematics formulas and signs following Indonesian language rules; 2) to make clear operational verbs in competency achievement indicators. The revisions are made as experts' suggestions.

To conclude, the overall expert validation results ensure that application mobile learning based on RMT approach is in valid criterion.

3.3.2 Electronic Worksheet (E-LKPD)

With the same learning objectives, developing student electronic worksheets, however, requires different organization considerations that are more activities to engage students. We also test the expert validation of E-LKPD: material validation, media validation, and language validation. The score results of the validation are an average score of 88,13 in very valid criteria for material validation, an average score of 94,62 in very valid criteria for media validation, and an average score of 93,64 in very valid criteria for language validation.

Some suggestions from the experts: 1) The video display should be enlarged so students can see the video clearly; images that have no meaning in learning should be removed so as not to confuse students; 2) The image on the cover should be adjusted to the content of the material, use contextual images that are appropriate to mathematical concepts; 3) Correction of inconsistent use of capital letters. Apart from that, experts' comment that the E-LKPD developed is good and can be understood well, and the layout and language are easy to understand.

From the expert validation results and revisions made considering experts' suggestions, it can be concluded that the E-LKPD developed-based RMT approach is a valid criterion.

3.3.3 Test Instrument

In line with mobile learning and E-LKPD, a test instrument is developed to assess the same learning objectives; however, the development of this RMT test instrument only discusses and presents questions at the qualitative thinking level.

The test instrument is validated by 3 experts: two lecturers of a mathematics education study program and a mathematics teacher in SMAN 4 Tanjungpinang. The validation aspects include content, construct, and language to measure the two packages of test instruments. The results of the validation questionnaires are presented in Table 5.

Table 5. Validation results of test instrument

Aspect	Average Score	Criteria
Content	89,2 %	Sangat Valid
Construct	90,2 %	Sangat Valid
Language	88,2 %	Sangat Valid

Based on the data analyzed, the two question packages of 10 questions with average scores are all in very valid criteria. Some suggestions by experts are followed up: to give references in some pictures used, to simplify some sentences in some questions, and to revise the test instruments.

Finally, the quantitative and qualitative data results show that the mobile learning, electronic worksheets, and test instruments are valid. Valid instruments in this study mean that mobile learning, worksheet, and test instrument have been constructed in a valid way that entails how RMT becomes the basis of development and are rooted in the learning process in the topic of sequences and series. Valid instruments are good sources for the learning process. However, a series of tests need to be carried out to ensure the quality of the instruments in other contexts, such as practicality and effectiveness.

4 Conclusion

The paper contributes to addressing students' weak understanding of the concepts of sequences and series and provides theoretical and practical examples of integrating the RMT approach in learning mathematics. The learning materials mobile learning, E-LKPD are developed for students in Class XI to learn sequence and series concepts. Materials delivery integrates 3 phases of RMT to help students in the process of constructing conceptual understanding. From the validation data results, it can be concluded that mobile learning and E-LKPD are very valid. Meanwhile, RMT test instrument is developed for the qualitative level of thinking. From the expert validation results, it can be concluded that the two packages of test instruments developed are very valid. This study is limited to the validation phase. Further research can be conducted to explore the effectiveness of the learning materials, and test instruments can be developed for the next 2 levels of cognitive. However, mobile learning that was developed and tested valid entails that the study can add to a new learning strategy that proposes technology and digital tools to engage students in the learning process and promote RMT approach.

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