Kinematic and force parameters of an elastic wheel when rolling on a flat rigid surface

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Abstract. The implementation of forces and moments applied to the wheels of a vehicle occurs in the zone of contact of the wheels with the supporting surface, and therefore the analytical determination of their force and kinematic parameters should be based on consideration of the phenomena occurring in contact. One of the directions for solving these problems is based on a technique based on the theory of preliminary displacement of wheel surface elements in the contact zone with the supporting surface and dividing the contact area into adhesion and sliding zones, which makes it possible to take a new approach to the consideration of wheel rolling mechanics. The use of this technique makes it possible to obtain relatively simple equations for such power and kinematic parameters of the wheel as the coefficient of rolling resistance, the coefficient of tangential elasticity of the wheel, and hysteretic losses in the wheel material. The derived equations contain a small number of parameters that can be easily determined experimentally and are convenient for both theoretical and practical use. Comparison of theoretical and experimental results with studies of other authors shows their relatively small discrepancy, which confirms the validity of the research methodology used, as well as the assumptions made. Keywords: wheel, rolling, rolling resistance, friction, hysteresis, tangential elasticity coefficient.

1 Introduction

Studies of wheel motion are based on various mathematical models that describe the process of its rolling [1-21]. At the same time, one can note the variety of approaches used to solve this problem, the results obtained, as well as the presence of inaccuracies and uncertainties inherent in a considerable number of studies.

Since the implementation of forces and moments applied to the wheel occurs in the contact zone of the wheel with the supporting surface, the analytical determination of the force and kinematic parameters of the wheel should be based on the consideration of contact phenomena occurring in the contact zone [16,22-34].

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It is convenient to consider the mechanics of interaction of a wheel with a supporting surface [16,23,33,34], using the so-called «reversed mechanism» of a friction pair «elastic wheel – supporting surface», in which the wheel mentally stops, while rotating around its axis at a speed ω , and the supporting surface moves in the direction opposite to the movement of the wheel at a speed «–V».

When the wheel rolls in contact with the supporting surface, tangential displacements of points on the running surface of the wheel $U_x = \xi_x(a - x)$, arise due to the implementation of a tangential force in the contact. These displacements in the traction area in contact are proportional to the relative loss of wheel speed

$$\xi_x = \frac{r_r^f}{r_r} - 1,$$

where r_r and r_r^f - rolling radius and free rolling radius of the wheel. The ratio $V/\omega = r_r$ is called the rolling radius. The radius of free rolling of the wheel corresponds to the case of wheel rolling in the absence of a tangential force in the contact.

As the coupled elements of the wheel and the base move in the contact zone, their tangential displacements increase; therefore, the tangential friction force acting between the coupled elements also increases. At the point of contact where the increased friction force reaches the maximum adhesion force, i.e.$q_t^{sl} = \mu q_n$a breakdown occurs and sliding occurs on the entire part of the contact located behind the breakdown point (fig. 1).

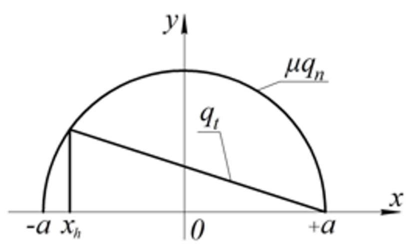

Fig 1. The accepted law of distribution of normal pressuresq_n (parabolic) иand tangential stresses q_t (linear) in the longitudinal direction of the contact; μ - is the sliding friction coefficient, a – is the half-length of the contact area, x_h- is the coordinate of the boundary of the adhesion and sliding areas.

With an increase in the loss of wheel speed and a corresponding increase in the tangential force acting in the contact, the sliding zone increases, as well as the power of friction losses in the contact, which characterizes the intensity of wear on the treadmill and the rolling resistance of the wheel.

2 Materials and methods

To find the main dependencies that determine the force and kinematic characteristics of an elastic cylindrical wheel during rectilinear rolling on a flat rigid surface, we use the methodology and results of the authors' previous research [23,33,34].

Comparing the formulas of E.A. Chudakov

$$r_r = r_r^f \pm \gamma_F F, \tag{1}$$

determining the dependence of the rolling radius of an elastic wheel on the longitudinal tangential force, with the dependence

$$r_r = r_r^f - \frac{2q_{n_0} r_r^f a}{3\lambda_x F_z} F_x, \quad (2)$$

obtained earlier in [33], it can be noted that they have the same structure. By equating the right-hand sides of equalities (1) and (2), we can obtain a formula that allows us to calculate the tangential elasticity coefficient included in the formula of E.A. Chudakova:

$$\gamma_F = \frac{2q_{n_0} r_r^f a}{3\lambda_x F_z}, \quad (3)$$

which, using the expressions given in [33], gives:

$$\gamma_F = \frac{r_r^f a}{2kr_d F_z s} \approx \frac{a}{2kF_z s}. \quad (4)$$

From the joint solution of the equations of power balance and force equilibrium of a wheel, the relationship between the moment applied to the wheel and the tangential force realized in contact can be represented in the form$M = F_x r_r^f$. This allows us to establish a connection between the coefficients$\gamma_F$and$\gamma_M$of tangential elasticity in the formula of E.A. Chudakov:

$$r_r = r_r^f - \gamma_M M = r_r^f - \gamma_F F_x.$$

From here

$$\gamma_M = \frac{\gamma_F}{r_r^f} = \frac{a}{2kr_d F_z s}. \quad (5)$$

The quantitie a and $r_d = r - W_0$ included in the last formulas can be determined experimentally or theoretically. In particular, to determine the deflection$W_0$of pneumatic tires, the dependencies presented in [31,35]; for solid rubber tires - formulas given in[36,37].

The half-length of the contact area of a rigid flat base and a wheel with a pneumatic or solid rubber tire is less than the value$a = \sqrt{r^2 - r_d{}^2}$, determined from purely geometric considerations (r –is the outer radius of the wheel, r_d– is the distance from the wheel axis to supporting surface).

Analysis of a large number of experimental values of the half-length of the contact area of automobile tires with a rigid flat base shows that, with an accuracy of 1...3%, the value of*a*can be calculated using the formula:

$$a = 0.85\sqrt{W_0(2r - W_0)} \approx 0.85\sqrt{2rW_0},$$

where W_0 – wheel deflection.

V.S. Nikitin [39] came to almost the same results, having experimentally found that for tractor tires:

$$a = (0.76 \ldots 0.84)\sqrt{W_0(2r - W_0)}\,.$$

The simple expressions obtained above allow us to calculate$\gamma_M$and$\gamma_F$with a fairly high accuracy.

To confirm this, table 1 provides calculated values of γ_M for a number of tires and values obtained based on a large series of experiments [31].

Table 1. Calculated values of γ_M for a number of tires and values obtained based on a large series of experiments

Parameter mm/decanewton · m	Tire		
	I-P184	OI– 25	I– 247
$\gamma_M^{exper.}$	0.05	0.067	0.031
$\gamma_M^{theor.}$	0.053	0.0636	0.034

A comparison of theoretical and experimental results shows their relatively small discrepancy, which confirms the validity of the assumptions made, as well as the research methodology used.

When determining power losses due to hysteresis in the tire material, we will take into account only the vertical deformation of the tire at the contact area under the influence of normal load (since the deformation of the wheel in the tangential direction is much less), which allows us to neglect power losses due to hysteresis in the tangential direction (although in practice their determination is not possible special difficulties):

$$P_h = 2b\beta_h \int_0^a q_n \left|\frac{dW}{dt}\right| dx,$$

where β_h - hysteresis loss factor, $\frac{dW}{dt}$- tire deformation rate in the normal direction:

$$\frac{dW}{dt} = \frac{dW}{dt}\frac{dx}{dx} = \frac{dW}{dx}V = \frac{dW}{dx}\omega r.$$

From geometric considerations it can be approximately assumed that:

$$W = \frac{a^2}{2r} - \frac{x^2}{2r} = \frac{a^2 - x^2}{2r} = W_0\left(1 - \frac{x^2}{a^2}\right),$$

where $W_0 = \frac{a^2}{2r}$- maximum wheel deflection (in the middle section).

The rate of deformation of the tire in the normal direction will be equal to $\frac{dW}{dt} = -\frac{x}{r}V$. Then the power loss due to hysteresis will be equal to:

$$P_h = 2bV\beta_h \int_0^a q_{n_0}(a^2 - x^2)\frac{x}{r}dx = \frac{3}{16}\beta_h a\omega F_z.$$

With the found value of P_h the moment due to hysteresis in the tire material is found as:

$$M_h = \frac{P_h}{\omega} = \frac{3}{16}\beta_h a F_z.$$

Knowing the moment due to hysteresis, we can also find the shift arm of the normal reaction of the base due to hysteresis:

Equating to this expression a similar expression $b_h = f_0 r_r^f$, derived by R.V. Virabov [16,22] for the same shoulder,

$$b_h = \frac{3}{16}\beta_h a = f_0 r_r^f$$

we get the formula

$$f_0 = \frac{3}{16}\frac{a}{r_r^f}\beta_h,(6)$$

which makes it possible to calculate by calculation the coefficient of rolling resistance of the driven wheel based on its parameters$a$andβ_h, obtained when the wheel is statically pressed to the base.

The hysteresis loss coefficient can be found using, for example, the technique described in [39].

The rolling resistance of a wheel is estimated by the rolling resistance coefficient, which is calculated as the ratio of the rolling resistance force$F_f$applied to the wheel axis to the normal reaction to the wheelF_n.

The rolling resistance force can be found from the wheel power balance equation:

$$F_f V = P_h + P_{fr}, \tag{7}$$

where P_h $and P_{fr}$ - power losses, respectively, for hysteresis and friction in contact, which are determined by the following formulas [23,34,40]:

$$P_h = F_n b_h \omega \quad P_{fr}^{dr} = F_x \xi^{dr} V_0 P_{fr}^{brake} = F_x \xi^{brake} V \tag{8}$$

where b_h- displacement of the normal reaction of the supporting surface, caused by hysteresis, relative to the wheel axis; ω — wheel angular speed; F_x — longitudinal force acting in contact of the wheel with the supporting surface; V — wheel axle speed, V_0 - the speed of the wheel axis during free rolling (when its axis is free from longitudinal force, and therefore the tangential force in contact is zero); ξ - relative loss of speed; the index «*dr*» refers to the drive wheel, the index «$brake$» - refers to the brake and driven wheels.
For the driven wheel, taking into account the above dependencies

$$F_{f_0} V = F_n b_h \omega + F_{f_0} \xi^{iw} V = F_n b_h \omega + F_{f_0} V - F_{f_0} \frac{r_r^f}{r_r} V, \tag{9}$$

where F_{f_0} —driven wheel rolling resistance force.

For the drive wheel, the rolling resistance force is determined from the equality

$$F_f V = F_n b_h \omega + F_x \xi^{dr} \omega r_r^f, \tag{10}$$

After transformations, taking into account the expressions for$b_h$andξ^{dr}, we obtain

$$f = f_0 \frac{r_r^f}{r_r} + \frac{F_x}{F_z} \frac{r_r^f - r_r}{r_r}. \tag{11}$$

The formula for determining rolling resistance during the braking mode of wheel rolling can be obtained in a similar way from the equation

$$F_f V = F_n b_h \omega + F_x \xi^{brake} V \tag{12}$$

$$f = f_0 \frac{r_r^f}{r_r} + \frac{F_x}{F_z} \frac{r_r - r_r^f}{r_r}. \tag{13}$$

If we consider the longitudinal force F_x for the brake wheel to be negative, then equation (13) will be common for all rolling modes.

Using the above dependencies, it is shown in [40] that the minimum value of the rolling resistance coefficient corresponds to the case when the wheel is driven simultaneously by a longitudinal force equal to half the force when the wheel is rolling in the driven mode, and a driving moment equal to half the rolling resistance moment from hysteresis (i.e. this is the case corresponding to the middle of the neutral mode).

3 Conclusions

Consideration of the mechanics of rolling of an elastic wheel, based on an analysis of the phenomena occurring in the contact of the wheel with the supporting surface, including the theory of preliminary displacement of the elements of the wheel surface in the contact zone with the supporting surface and the division of the contact area into adhesion and sliding zones, made it possible to obtain relatively simple equations for such power and kinematic parameters of the wheel as the coefficient of rolling resistance, the coefficient of tangential elasticity of the wheels, and the power of hysteresis losses. The derived equations contain a small number of parameters that can be easily determined experimentally and are convenient for both theoretical and practical use.

Comparison of theoretical and experimental results with studies of other authors shows their relatively small discrepancy, which confirms the validity of the assumptions made, as well as the research methodology used.

References

1. J. Wong, Theory of land vehicles, Mechanical Engineering.Moscow (1982)
2. F. Kosar, *Wheeld armoured reconnaissance vehicles.* Armada, **2,** 36-56 (1984).
3. Gh. Apetrei, D. Velicu, B. Bobancu, *Model petru analisa neconcordantei cinematice din sistemue de rulare al autotrenului cu semiremorca motoare la derlasarea in panta.* Bul.Univ. Brascv**, 20-A**, 97-104 (1978).
4. M.Yu.Karelina, T.A.Balabina, V.V.Filatov, A.N. Mamaev, A. S. Vorontsov, IOP Conference Series: Materials Science and Engineering **832**, 012079 (2020).
5. I. Brylev, S. Evtiukov, S. Evtiukov, Transportation Research Procedia **36**, 84–89 (2018).
6. S. Evtiukov, E. Golov, G. Ginzburg, Transportation Research Procedia **36**, 157–165(2018).
7. S. Repin, S. Evtiukov, S. Maksimov, Transportation Research Procedia **36**, 661–668 (2018).
8. S.K. Clark, Tire rolling resistance. Ed. Rubber Div, ACS (1983).
9. H.B. Huang, Y.J. Chiu, C. Wang, X. Jin, Proc. Inst. Mech. Eng. Part D J. Automob. Eng., **229**: 197–213 (2015).
10. E. Bakker, L. Nyborg, H.B. Pacejka, *Tyre modeling for use in vehicle dynamics studies.* SAE Technical Paper 870421 (1987).
11. S. Palanivelu, K.V.N. Rao, K.K. Ramarathnam, Mech. Syst. Sig. Process. **64:** 385–402 (2015).
12. W. Liang, J. Medanic, R. Ruhl, Veh. Syst. Dyn. **46**, 197–227 (2008).

13. Y. Wang, M.V. Blundell, G. Woodand, C. Bastien, Proc. Inst. Mech. Eng. Part K: J. Multi-body Dyn. **228**, 400–413 (2014).
14. H. Wang, I.L. Al-Qadi, I. Stanciulescu, Int. J. Pavement. Eng. **13:** 310–321 (2012).
15. M. Behroozi, O.A. Olatunbosun, W. Ding, Mater. Des. **35**: 810–819 (2012).
16. R.V. Virabov, *On the implementation of tangential force in the contact zone of elastic bodies during rolling*, Mechanical engineering **2** (1967)
17. R. Lorenz, Schiene und Rad. VDJ-Z 2, **6** (1928).
18. G. Freudenstein, Luftreifen bei Schrag und Kurvenlauf (Experimentalle und theoretische Untersuchung an LKW-Reifen). Deutdche Kraftfahrtforschung, H. 152 (1961).
19. E. Fiala, Seitenkrafte am rollendenLuftreifen. VDJ-Z, Bd. 96, ¹29, S. 973...79 (1954). TranslationofTsPBNo. 17347, Moscow (1956).
20. T.A. Balabina, Y.I. Brovkina, A.N. Mamaev, Lecture Notes in Mechanical Engineering **9783319956299**, 2027-2035 (2019).
21. Xianbin Du, et.al., J. Braz. Soc. Mech. Sci. Eng**.** **39:** 3315-3327 (2017).
22. R.V. Virabov, Mechanical engineering **4,** 78-85 (1967).
23. R.V. Virabov, A.N. Mamaev, Machine mechanics. Moscow.Science. **57**, 101-105 (1980)
24. M.Y. Karelina, T.A. Balabina, A.N. Mamaev, *Mechanics of Elastic Wheel Rolling on Rigid Drum.* Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019). Lecture Notes in Mechanical Engineering, 531-540 (2019).
25. J. Halling, Jour. of Mech. Eng. Science **6,** 64 (1964).
26. A.N. Mamaev, T.A. Balabina, I.V. Odinokova, V.V. Gaevskiy. IOP Conference Series: Materials Science and Engineering **632**, 012083 (2019).
27. B.D.A. Phillips, The Science and Technology. V.**4,** VIII, ¹3, 155-158 (1976).
28. T.A. Balabina, D.S. Simonov, V.R. Rogov, A.N. Mamaev, A.S. Vorontsov. IOP Conference Series: Materials Science and Engineering **832**, 012077 (2020).
29. M.Yu.Karelina, T.A.Balabina, T.Yu.Cherepnina, A.N. Mamaev, A. S. Vorontsov. IOP Conference Series: Materials Science and Engineering **832**, 012078 (2020).
30. S. Kim, K. Kondo, T. Akasaka, Tire Sci. Technol. **28:** 2–32 (2000).
31. V.N. Evgrafov, et.al., Automotive industry **8**, 16-17 (1982)
32. R.V. Virabov, *Traction properties of friction gears*, Mechanical engineering, p.263 (1982)
33. A. Mamaev, T. Balabina, M. Karelina, E3S Web of Conferences **376** (2023)
34. R.V. Virabov, A.N. Mamaev, *Determination of the power of friction losses in the contact of a friction pair - a wheel with a pneumatic tire - a rigid base*, Interuniversity collection of scientific papers “Stepless variable transmissions”. Yaroslavl, p.61-67 (1978)
35. V.N. Evgrafov, *Study of the influence of the working qualities of a wheel propulsion device on some technical indicators of a road train.* Abstract of the dissertation for the academic degree of Ph.D. Moscow (MAMI. -1979)
36. A.N. Mamaev, *The influence of the design parameters of elastic wheels on the amount of their deflection and the dimensions of the contact area with a rigid base*, Interuniversity collection of scientific papers “Safety and reliability of the car.” Moscow. (MAMI. -1982)

37. A.N. Mamaev, E.A. Alepin, *Determination of the dimensions of the contact area and deflection of a wheel with a rubber tire when the wheel is statically pressed against a rigid base*, Collection of scientific works “Mechanical Science” **251**, 82-85 (1980)
38. V.S. Nikitin, *The influence of wheel load conditions on traction performance and wear of tractor tires*, Abstract of the dissertation for the scientific degree of Ph.D. Moscow. (MGMI. 1989)
39. A.N. Mamaev, *On determining the hysteresis loss coefficient of highly elastic rolling elements,* Moscow. NIINavtoprom **779**. ap-D82 (1982)
40. A. Mamaev, T. Balabina, M. Karelina, E3S Web of Conferences **376**, 01065 (2023)