

Evaluating Performance: A Study of Encrypted vs. Unencoded Signals in SISO-OFDM with LS and MMSE Estimations

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Abstract: The continuous progress in digital communication has played a crucial role in meeting the increasing need for faster data rates. Orthogonal Frequency Division Multiplexing (OFDM), a pivotal methodology in this progression, attains improved data rates through the efficient utilisation of densely packed carriers within a specified channel bandwidth. This article focuses on the investigation of channel estimation in OFDM systems. the study of search lies in its examination of the performance consequences associated with the incorporation or lack thereof of a convolutional encoder in OFDM systems and looks at how well two well-known channel estimation algorithms, Least Square (LS) and Minimum Mean Square Error (MMSE), work in 4-Quadrature amplitude modulation (4-QAM) OFDM systems with and without a convolutional encoder with a comprehensive evaluation of the efficacy of the OFDM system across various channel conditions. It uses MATLAB implementations as its main tool. The findings of the study indicate that the MMSE algorithm, despite its higher complexity, exhibits superior performance in comparison to the LS algorithm when combined with a convolutional encoder. The gain in terms of bit error rate (BER) improvement approximately 12 dB. This represents the logarithmic scale improvement in BER from BER1(uncoded) to BER2(coded) at the same the energy per bit to noise power spectral density ratio (Eb/N0) of 40 dB.

1 Introduction

High performance, high capacity, and high bit rate are becoming more and more in demand in the field of wireless communications. To achieve these goals, digital approaches were used.[1]. Orthogonal Frequency Division Multiplexing (OFDM) is a digital modulation technique that enables the efficient transmission of high data rates over channels with comparatively little complexity. lately, there has been a notable increase in the use of this technology in wireless local and network systems. It has grown up as an important component of well-established standards including Long-Term Evolution (LTE) standards, WiMAX, WIFI, Digital Audio Broadcasting (DAB) and 3GPP. [2,3] OFDM is a modulation technology that employs the principle of sub-stream spectrum duplication. In this approach, each sub stream is broadcast orthogonally over a dedicated sub-channel, allowing for their subsequent separation in a demodulator [4] This approach is implemented to improve the system's ability to resist frequency-selective fading and narrowband interference.[5]. It is often used in several communication systems is widely preferred because of its capacity to provide high-speed performance while effectively mitigating the issues of Inter-Carrier Interference (ICI) and Inter-Symbol Interference (ISI) [6] An essential determinant in the process of data transmission is the estimation of the channel. This estimate is crucial before demodulating OFDM signals because each mobile communication system has its own set of time-varying factors and frequency-selective fading that can affect the channel. The estimation channel is established primarily through the insertion of pilot symbols into specific subcarriers of each OFDM symbol or into all subcarriers of an OFDM symbol. the methodology investigated in this research is comb-type based channel estimation, which utilises pilot symbols strategically positioned on particular subcarriers of every OFDM symbol. This methodology is distinguished by its strategic application of time domain interpolation, which integrates linear and spline cubic interpolation methods. The primary emphasis of this manuscript is a comparative analysis and practical implementation of the Minimum Mean-Square Error (MMSE) and Least Square (LS) Estimators [7] in the context of channel estimation for the comb-type configuration under consideration. The distinctive aspect of this study resides in its investigation of these estimators in the context of OFDM systems, more precisely by employing 4-Quadrature amplitude modulation (4-QAM) modulation.

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This study assesses the estimation of comb-type channels in 4-QAM modulated OFDM systems [8] by comparing the performance of LS and MMSE algorithms in Rayleigh fading multipath channels, with and without a convolutional encoder [9]. In Section II, the fundamental OFDM system model is described. In Section III, the estimation procedure for a Rayleigh fading multipath channel, employing a comb-type prototype arrangement, the results of the simulation parameters are examined in Section IV.

2 Describing the OFDM system

The complexities of the OFDM system will be clarified in two separate scenarios: the first scenario assumes that encryption is not utilised, and the second scenario incorporates Convolutional Coding when used with OFDM.

2.1 Uncoded OFDM

In the OFDM system, as depicted in Fig.1, the transmission process initiates with the mapping of information bits to their respective data symbols. These symbols may be derived from a variety of modulation techniques such as Quadrature Phase-Shift Keying (QPSK), QAM, with this particular system employing 4-QAM. Post mapping, the data symbols are distributed into M parallel streams, ensuring even spacing for optimal transmission efficiency. Following this, pilot symbols are inserted within these streams for enhanced channel estimation and signal integrity. Finally, the arrangement of symbols is fed into an Inverse Fast Fourier Transform (IFFT), Cyclic Prefix (CP) were extended as illustrated in Fig.2 by copying the final Ng samples and pasting them in front, The channel length must be shorter than the cyclic prefix. Regarding OFDM systems which effectively transforms the serially transmitted signal into parallel orthogonal signals, thereby completing the transmission preparation phase in the OFDM system.[10]. Then, it is transmitted over the specified channel. given as equation (1) [11].

$$y(s) = x(s) \otimes G(l) + n(s) , s = 0,1,2, \dots, N - 1 \quad (1)$$

Where: $y(s)$: is the received signal, $x(s)$: is the transmitted signal, $G(l)$: is the Channel Impulse Response (CIR), $n(s)$: is represent the AWGN noise and N : sub-carriers

At the receiver, following serial-to-parallel conversion, CP removal is performed to eradicate inter-symbol interference. Following this, a Fast Fourier Transform (FFT) is applied to the signal in order to distinguish the orthogonal subcarriers. Channel estimation is accomplished through the utilisation of inserted pilot symbols and methods including LS and MMSE. Following equalisation to rectify channel effects, pilot symbols are eliminated. A parallel-to-serial conversion concludes by preparing the signal for the last demodulation step.[12].

The received data is equalised utilising the frequency domain equaliser after channel estimation accordance with the following equation (2) [11]:

$$\hat{X}(s) = \frac{Y(s)}{\hat{G}(s)} \quad (2)$$

Where: $y(s)$: is the received signal, $x(s)$: is estimation of transmitted signal, $\hat{G}(s)$: is estimated of CIR.

Input denotes the frequency domain response of the channel. Frequency domain equalisation is a valuable technique for rectifying the fading of symbols caused by multipath. Discussion of the results appears in Section V.

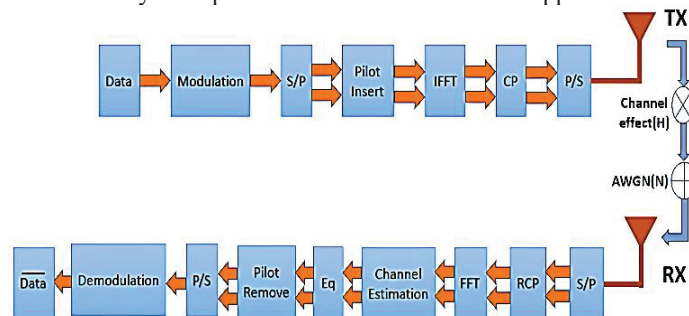


Fig. 1. Uncoded OFDM Transceiver Systems

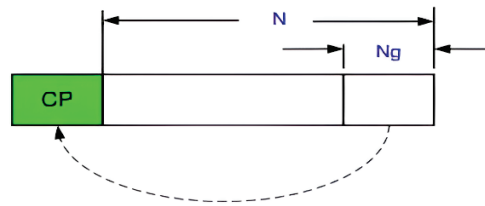


Fig. 2. cyclic prefix adds

2.2 Coded OFDM

To enhance the existing framework, encryption and interleaving modules have been integrated into the enhanced system architecture. Encryption is applied to the transmitted signal, thereby enhancing its protection against unauthorised interception. Following this, an interleaver is implemented to distribute the data bits throughout the transmission sequence in order to reduce the impact of burst errors. In a similar fashion, the dispersed bits are reordered to their original sequence at the receiver end through the use of deinterleaving. Subsequently, decoding algorithms are implemented to decrypt the received data, thus restoring the information to its initial state. The security and integrity of the data transmitted through the OFDM communication system are both guaranteed by this enhanced system. The following Fig.3 shows the structure of the system after adding the encoder and interleaver.

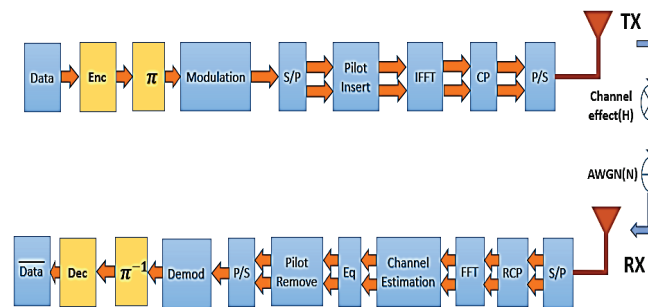


Fig. 3. Coded OFDM Transceiver Systems

2.3 Convolutional code

Convolutional codes are a crucial element in the field of error-correcting codes, especially in systems that often suffer faults, which refer to a consecutive series of erroneous bits. These codes have been derived from convolution processes often used in mathematics and signal processing. They have shown to be very valuable in communication systems, as they provide protection for data against the challenges posed by noise and interference during transmission. In contrast to block codes, which divide the input data into blocks and introduce redundancy to each block, convolutional codes use a distinct strategy by using memory and analysing a continuous stream of data bits in conjunction with their previous bits to generate the encoded sequence.

convolutional code is a type of channel coding that is used to transmit the input signals through an unreliable channel [13,14]. Convolutional code is implemented using shift registers and adders as shown in Fig.4. The first shift register represents the input bits, while the next two shift registers-explain the states of the convolutional code. As the Figure illustrates, there are two outputs X1 and X2 while the encoder output gets switched between them. The coded signal using convolutional code is characterized by the set of the analytical representation (n, k, m) where:

- n: is the number of outputs
- k: is the number of bits in each shift register
- m: is the number of the shift register

In addition, the code rate will be represented as (k/n). Therefore, the representation of Fig.4 will be (2,1,3) and the code rate is equal to (1/2).

Moreover, three methods are used to describe the convolutional code: tree diagram, state diagram, and trellis diagram.

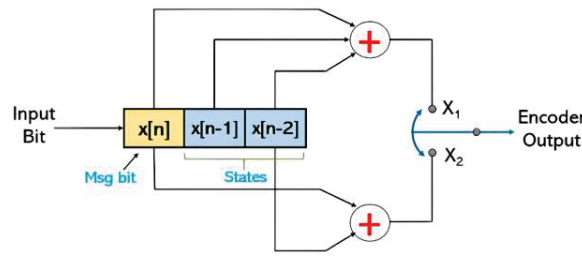


Fig. 4. The block diagram of the convolutional code

At the receiver end, the convolutional decoder employs two distinct kinds of decoders. The first decoder is the Maximum Likelihood (ML) sequence decoder, which is realized by the use of the Viterbi algorithm. The second approach is the MAP, which is implemented by the BCJR algorithm, called after its inventors: Bahl, Cocke, Jelinek, and Raviv [15].

The Viterbi decoder is used for our simulation because of its comparatively lower complexity when compared to BCJR. However, their performance is practically identical. Given the intricate nature of these algorithms, their elucidation is confined to the reference [16].

3 Descriptions of the Channel Estimation System

The receiver's most important phase is channel estimation. To estimate the channel coefficients, which are subsequently used in the equalizer step to get the transmitted signal, it employs one of the channel estimation approaches. Different channel estimate methods are being taken into account in relation to various elements of the implementation of the OFDM system. In addition to computing complex and needed performance, channel changes in time must be taken into account when estimating wireless channel coefficients. Pilot-based estimation of channels is the best and most extensively used technique. [7]. After adding a pilot to the transmitter, in receiver using the MMSE and LS to estimator.

3.1 Least Square LS

This method of channel estimation is the simplest; it involves interpolating the predicted channel at the pilot subcarriers and recovering the transmitted data from the resultant CIR. [17] The channel estimate \hat{G} is discovered using LS estimation technique After the process of simplification and derivation, for each subcarrier can be written \hat{G}_{LS} as equation (3) [12].

$$\hat{G}_{LS}[s] = \frac{Y[s]}{X[s]}, s = 0, 1, 2, \dots, N - 1 \quad (3)$$

$\hat{G}_{LS}[s]$: channel estimate use LS, $y(s)$: is the received signal, $x(s)$: transmitted signal, The simplicity of the LS method is a benefit, and the LS estimators are computed with a very low level of complexity without the need of any channel statistics information.

3.2 MMSE

MMSE channel estimation is a statistically rigorous method that aims to reduce the mean square error between the estimated channel and the real channel among the many channel estimation strategies. MMSE criteria is the foundation of MMSE channel estimation. This statistical criterion tries to identify the channel estimate that reduces the anticipated value of the squared difference between the estimated channel response and the actual channel response. It is based on the concepts of linear estimation theory. [18].

Consider Equation following gives the received signal's FFT as equation (4) [11]:

$$Y(s) = X(s)G(s) + n(s) \quad (4)$$

Let, the mean of channel coefficients is given by $E\{G(s)\} = 0$ and variance equation (5) [19]:

$$E\{n(s)|^2\} = N\sigma_n^2 \quad (5)$$

Where, L is number of multi-paths and N is number of sub carriers σ_n σ_g are channel and noise variances respectively. Let us assume that channel coefficients and AWGN noise are independent. Hence, equation (6) [19]:

$$E\{G(s)n^*(s)\} = E\{n(s)G^*(s)\} = 0 \tag{6}$$

then, the MMSE estimation for the s^{th} subcarrier is conventionally represented as follows: sub carrier is given by equation (7) [19]

$$\hat{G}_{MMSE}(s) = R_{G(s)Y(s)} R_{Y(s)Y(s)}^{-1} Y(s) \tag{7}$$

Where $R_{G(s)Y(s)}$ is covariance of $G(s)$ and $Y(s)$ can give in [19]

The rest of the derivation is in [19] to obtain the solution to the MMSE algorithm as equation (8) [19]

$$\hat{G}_{MMSE} = R_{GG} \left[R_{GG} + (XX^G)^{-1} N\sigma_n^2 \right]^{-1} \hat{G}_{LS} \tag{8}$$

This is the solution for MMSE channel estimation.

Both estimators are subject to distinct limitations. The MMSE typically exhibits a significant level of complexity, whereas the LS estimator is afflicted with a substantial Mean-Square Error (MSE).

4 SIMULATIONS AND RESULTS

This section provides an overview of the results obtained from the simulations conducted in accordance with the mathematical formulations and theoretical concepts described in Sections II and III, for simulating a fundamental OFDM system, the parameters listed in Table 1 were utilised.

Table.1. Simulation Parameters

Parameters	Value
Total number of sub-carriers (NFFT)	128
Total number of symbols per resource block (number of OFDM symbol) N_s	14
Number of used sub-carriers (N_{sc})= N_{fft}	128
Cyclic Prefix (N_{cp})	16
Pilot spacing (N_{ps})	4
Modulation type(M-QAM)	4-QAM
Channel Model	AWGN, Multipath channel
No of Taps	7
code rate of convolutional encoder	1/2

Fig. 5 provided illustrates a comparative analysis of convolutional encoder-free OFDM system channel estimation techniques.

The Bit Error Rate (BER) is calculated as a function of the energy per bit to noise power spectral density ratio (E_b/N_0). Under the tested conditions, the LS estimation with linear interpolation produces the maximum BER, indicating a less precise channel estimation. The application of spline cubic interpolation in LS estimation yields an improvement, as indicated by a reduced BER, which implies enhanced error performance and channel representation. The BER of MMSE estimation algorithm is the lowest throughout the entire E_b/N_0 range, which is in close proximity to the optimal performance predicted by the established channel curve. The efficacy of MMSE in error mitigation and channel information extraction is demonstrated by these results, despite the increase in computational complexity that this entails. The illustrated patterns in performance serve as valuable standards against which to evaluate channel estimation approaches in OFDM systems, striking a balance between ease of implementation and error reduction.

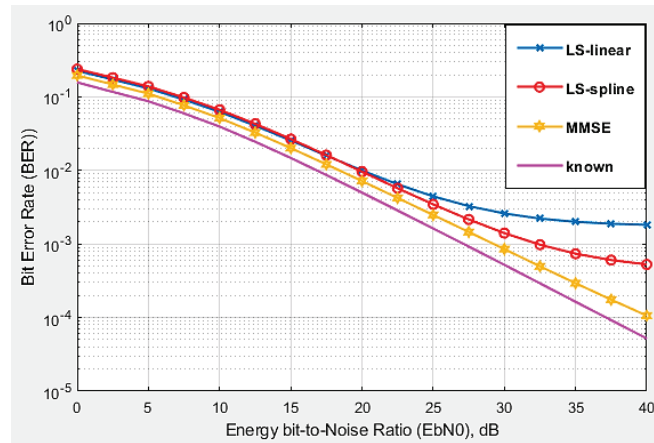


Fig. 5. BER vs EbN0 Uncoded OFDM with LS/MMSE Estimator

Fig.6 depicts the MSE performance of three channel estimation algorithms across varying Eb/N0 levels in an OFDM system without encryption. The MSE reveals the accuracy of channel estimations; a lower MSE denotes a more precise estimation. Among the three estimation methods, the 'LS-linear' estimation exhibits the maximum MSE across the board, indicating its lowest accuracy. The consistently reduced MSE of the 'LS-spline' estimation compared to the 'LS-linear' estimation suggests that spline interpolation provides a more accurate means of estimating the channel. The 'MMSE' algorithm demonstrates its superior estimation accuracy by outperforming both LS estimations with the lowest MSE across all Eb/N0 values. It is important to acknowledge, nevertheless, that the enhanced precision of MMSE resulting from the reduction in MSE is accompanied by an increase in computational complexity. This succinct comparison provides significant insights that can be applied to the design of systems, highlighting the inherent compromise between the complexity of algorithms and the accuracy of estimations.

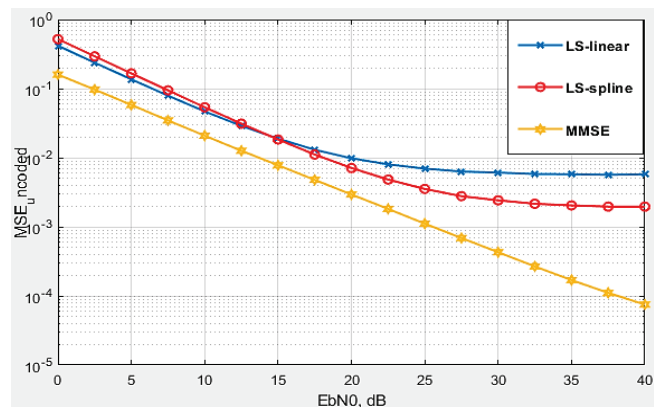


Fig. 6. MSE versus EbN0 for the LS and MMSE Estimators in Uncoded OFDM

The BER performance of a SISO-OFDM system utilizing a convolutional encoder is charted against varying Eb/N0 levels in Fig.7, delineating the comparative efficacy of different estimation algorithms. The LS-linear with convolution (LS-linear-c) algorithm yields the highest BERs, whereas the LS-spline with convolution (LS-spline-c) offers moderate improvements. Significantly, the MMSE with convolution (MMSE-c) algorithm excels, especially at an Eb/N0 of 30 dB, presenting a BER close to 10^{-4} . This trend of superiority extends to an Eb/N0 of 40 dB, where the MMSE-c algorithm demonstrates an even lower BER, indicative of its robust channel estimation capability when combined with convolutional encoding, though the exact value at 40 dB is 10^{-5} . It's evident that the MMSE-c algorithm approaches the ideal known channel (known-c) performance, with the latter at roughly 10^{-4} at 30 dB Eb/N0, underscoring the substantial benefits of MMSE-c in enhancing error performance in coded OFDM systems.

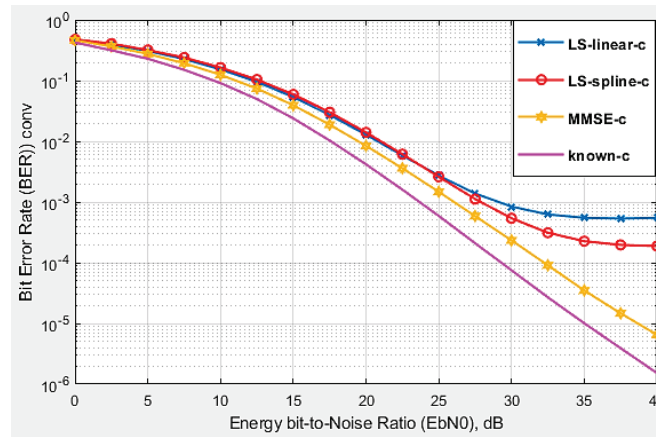


Fig. 7. BER vs EbN0 coded OFDM with LS/MMSE Estimator

Fig.8 exhibits the MSE performance of three channel estimation methods utilising a convolutional encoder in an OFDM system. A measure of the precision of a channel estimator, the MSE, decreases as the Eb/N0 increases for all methods. The LS-spline has a substantial reduction in MSE, which indicates improved channel estimation, whereas the LS-linear estimation produces the maximum MSE, which indicates reduced estimation accuracy. Particularly at higher Eb/N0 values, the MMSE estimation method outperforms both LS methods by obtaining the lowest MSE and, consequently, the maximum estimation accuracy. With a convolutional encoder and the MMSE estimator, the performance seen so far shows that this combination gives the most reliable channel estimation for an OFDM system.

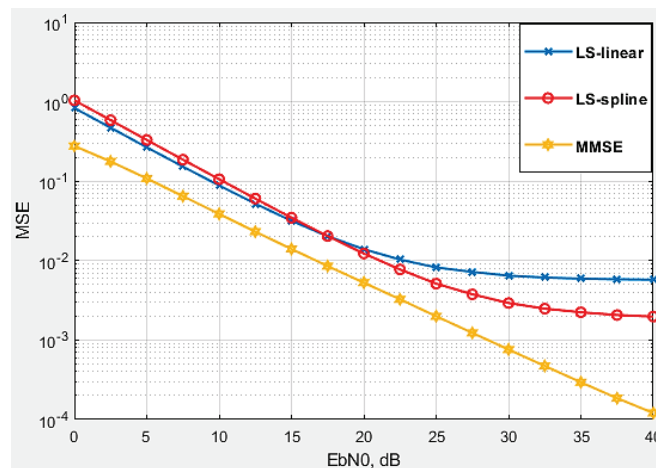


Fig. 8. MSE versus EbN0 for the LS and MMSE Estimators in coded OFDM

Fig.9 The overall clearly illustrates that the MMSE algorithm outperforms the known channel estimation in OFDM systems without convolutional encoding, as evidenced by its lower BER compared to OFDM systems with convolutional encoding. The exceptional outcome suggests that when the MMSE algorithm is used in conjunction with encryption methods, it not only preserves but also improves data integrity, exceeding the error performance of the unencrypted benchmark. The aforementioned results highlight the efficacy of MMSE in encrypted settings, providing significant enhancements in the dependability and resistance to errors of secure OFDM communications.

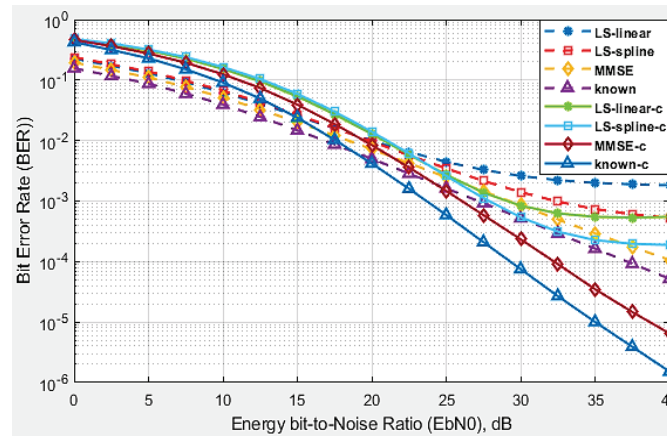


Fig. 9. Overall comparison BER vs EbN0 for the LS and MMSE Estimators with coded uncoded OFDM system

The gain from use Convolutional code in dB represents the logarithmic scale improvement in BER from BER1 (without coded) to BER2 (with coded) at the same of 40 dB, The gain in MMSE terms of BER improvement, is approximately 12.14 dB, in LS-linear approximately 5.21 dB and 4.48 dB in LS-spline. As for Known signal the gain is approximately 15.33 dB.

5 CONCLUSION

With and without a convolutional encoder, this study examines a channel estimation strategy that employs pilot-assisted comb-type training symbols and MMSE and LS algorithms. Channel estimation is an essential component in the design of OFDM systems, as it deals with the numerous signal distortions that occur during transmission, including reflection, refraction, and diffraction. Furthermore, in order to recover the initial transmitted signal, it is crucial that these channel distortions be efficiently nullified at the receiver, as the channel's variability is caused by mobility. A comparison of channel estimation methods in OFDM systems reveals that the MMSE algorithm offers the most precise channel estimation, resulting in substantial reductions in both MSE and BER at different SNR levels. Significantly, the MMSE algorithm accelerates the system's performance in comparison to the unencrypted benchmark when used in conjunction with a convolutional encoder. This underscores the importance of the encoder in enhancing error resilience and data integrity. The findings of this study support the notion that OFDM systems would benefit from MMSE with convolutional encoding for optimal performance, especially in situations where dependability and security are of the utmost importance.

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