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Co r-Equality Domination in Graphs

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Abstract

There are a lot of studies and scientific researches on domination and its different types. We introduce new types of domination. D is known as a co-r-equality dominating set because $\mathrm{D} \subseteq \mathrm{V}(\mathrm{G})$, such that$\forall\, \mathrm{v} \in \mathrm{V}-\mathrm{D}$, where r vertices in D dominate the vertex v, The cardinality minimum of all $\mathrm{MC^rEDS}$ is known as the co-r-equality dominance number, as shown by $\gamma_{\mathrm{cre}}(\mathrm{G})$. We determine best possible upper and lower bounds for $\Upsilon_{\mathrm{crs}}(\mathrm{G})$, discussed for several standard graphs such as : complete, complete bipartite, wheel graphs.

1.Introduction

Graph theory a fundamental structure in mathematics used to explain relationships between entities here can be found in [1, 2, 3, 4] . One of the most fundamental subjects within discrete mathematics [5]. A graph is a representation of point or nodes (vertices) connected by lines (edges) [6]. The subject has links to many branches of mathematics, including topology, algebra, probability and numerical analysis. We need graphics in our real life, such as cities, street, service agencies, and homes. In fact, dominance refers to the set of segment that govern all vertices (edges) in graph [7]. There for, vertices or edges can dominate the graph. Studying the issues of one of the topics of graph theory that is expanding the fastest is dominance. It is widely used in traffic jams, communication network, war planning, coding theory, social media, DNA analysis electrical communications and more,[18,19,20]. The importance of dominance in different applications gives rise to different types of dominance for this purpose[8, 9, 10, 11]. Dominant parameters are formed by imposing conditions on the dominant set, or by imposing condition on the outside of the dominant set, there are also definitions that include both methods. Here, the co-r-equality domination model of graph dominance is shown. There are limits on the co-r-equality domination number related to a graph's order, size, minimum degree, maximum degree, and other attributes. The co-r-equality dominance is calculated for a few modified and known graphs.

2. Co r-Equality Domination

Definition 2.1. Let G be a graph of order n and $D \subseteq V$ such that $\forall\, v \in V-D$, there are r vertices in D dominate the vertex v, so D is called a co-r-equality dominating set (C^rEDS).

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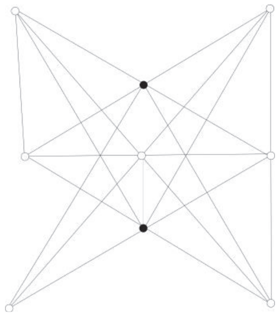
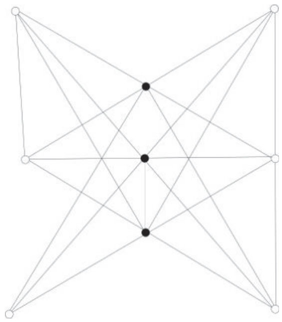


A

Figure 1. (A) graph G along with $\gamma(G)$=2



B

(B) graph G with $\gamma_{cre}(G)$=3.

Definition 2.2. Let $D \subseteq V$ be a CREDS in G, then D is minimal C^rEDS (MC^rEDS) if D has no proper CREDS. The cardinality minimum of all MC^rEDS is referred to as the co-r-equality dominance number and is represented by $\gamma_{cre}(G)$.

Properties 2.3. Let G be an order n graph and $D \subseteq V$ be C^rEDS, then

1) Each an isolated vertex belong to every C^rEDS.

2) $s \leq \gamma_{cre}(G) \leq n$, where s is the number of isolated vertices.

3) $deg(v) \geq r, \forall v \in V - D$

4) If G is C^1EDS and D and $V - D$ are a separate group and S be the collection of isolated vertices, then the induced sub graph produced by $G - S$ is isomorphic to perfect matching sub graph.

5) If a graphic G contains an isomorphic spanning subgraph to star graph, then

$\gamma_{cre}(G) = 1$.

6) Let G is a graph that is connected and $r = |D|$, so, If $\gamma_{cre}^{-1}(G) = |V - D|$, then there is an induced sub graph H such that $H \equiv K_{m,n}$.

7) If G has w components, then $\gamma_{cre}(G) \geq w$.

Proof. 1, 3, 5 and 7) It is obvious that by definition of CREDS.

2) Depending on 1, the lower bound holds. Moreover, if $G \equiv N_n$, then $\gamma_{cre}(G) = n$, so the upper bound is hold too.

4) Since G is C^1EDS, then $\forall v \in V - D$ is dominated by exactly one vertex. Moreover, since D and $V - D$ are an independent set, so $G - S$ is isomorphic to perfect matching sub graph.

6) Since $r = |D|$, then each vertex in the collection that is adjacent to every other vertex D in the set $V - D$. Additionally, if $\gamma_{cre}^{-1}(G) = |V - D|$, then each vertex in the collection that is adjacent to every other vertex D in the set $V - D$ (as an example, see Figure 2.). Thus, the result is obtained

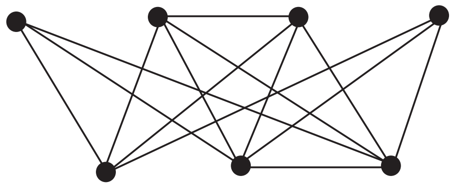


Figure 2.

Proposition 2.4. Let G be a graph with size m and order n, , and $D \subseteq V$ is C^rEDS, then

$r\ |V - D| \leq m \leq \frac{n(n-1)}{2}$, where m is the number of edges.

Proof. Let G be a graph of order n, size m, since D is CREDS, then, each vertex in the set $V - D$ is near r vertices in the set D. Now, if $\langle D \rangle$ and $\langle V - D \rangle$ are independent induced subgraphs, then the lower bound is getting. The upper bound is satisfied when the graph G is a complete. Thus, required is obtained.

Observation 2.5. If $G \equiv K_n,\ S_n\ or\ W_n$, then $\gamma_{cre}(G) = 1$.

Proof. Straightforward from Proposition 2.1(5).

Proposition 2.6. If $G \equiv P_n,\ C_n$, then $\gamma_{cre}(G) = \left\lceil \frac{n}{3} \right\rceil$.

Proof. The following three cases depend on the value of n in module 3.

Case 1. If $n \equiv 0(mod\ 3)$, thus, let $D = \left\{v_{i+3k}, k = 0,1, \dots, \frac{n}{3} - 1\right\}$, clearly, There is only one vertex adjacent to each vertex in the set $V - D$, indicating that set D is the dominating set. Then, $\gamma_{cre}(G) = \frac{n}{3}$.

Case 2. If $n \equiv 1(mod\ 3)$, then let $D = \left\{v_{i+3k}, k = 0,1, \dots, \left\lceil \frac{n}{3} \right\rceil - 2\right\}$, Clearly, the set D is CREDS of the induced subgraph generated by the set $\{v_i, i = 1, \dots, n-1\}$. The remain vertex not dominated by the set D is $\{v_n\}$, so $D \cup \{v_n\}$ is CREDS. Therefore, the outcome is attained.

Case 3. If $n \equiv 1(mod\ 3)$, then let $D = \left\{v_{i+3k}, k = 0,1, \dots, \left\lceil \frac{n}{3} \right\rceil - 2\right\}$, Clearly, the set D is C^rEDS of the induced subgraph generated by the set $\{v_i, i = 1, \dots, n-2\}$. The remain vertex not dominated by the set D is $\{v_{n-1}, v_n\}$, so $D \cup \{v_{n-1}\}$ is C^rEDS. Thus, the outcome is attained.

From every instance listed above, the evidence is complete.

Proposition 2.7. Assume that G is a complete bipartite graph $K_{n,m}; n \leq m$, then

$$\gamma_{cre}(G) = \begin{Bmatrix} 1, & if\ n = 1 \\ 2, & otherwise \end{Bmatrix}$$

Proof. The $K_{n,m}$ consists of two disjoint independent sets $X = \{v_i, i = 1, \dots, n\}$ and $Y = \{v_i, i = 1, \dots, m\}$, hence there are the following two cases:

Case 1. If $n = 1$, thus, the graph $K_{1,m}$ isomorphic to star graph and by Proposition 2.1(5), the result is obtained.

Case 2. If $n, m \geq 2$, thus, let $D = \{v_i, u_j\}$ is C^rEDS.

Thus, from all cases above, the result is obtained.

Note 2.8. The reader can be notice that in Observation 2.5, Proposition 2.6, and Proposition 2.7 the $\gamma_{cre}(G) = \gamma(G)$, but this is not a fixed rule and as a counter example we take the following example that illustrate in the following figure

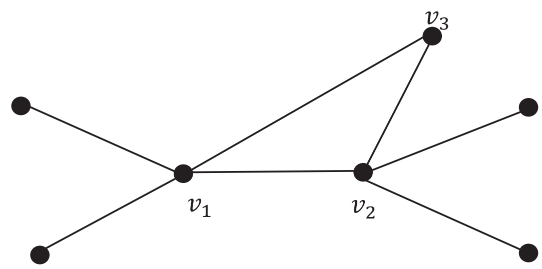


Figure 3

In the above figure, the set $D = \{v_1, v_2\}$ is a minimum dominating set, so $\gamma(G) = 2$, but it is does not CREDS, since the vertex $v_3 \in V - D$ and next to two of the set's vertices D while only one vertex of the set D is adjacent to the other vertices in the set $V - D$. Now, let $D_1 = \{v_1, v_2, v_3\}$ it is obvious that D_1 is a CREDS and $\gamma_{cre}(G) = 3$. Thus, in this case $\gamma_{cre}(G) \neq \gamma(G)$.

Theorem 2.3. If there are three vertices have $n - 3$ degrees $\{u, v, w\}$, then $\gamma_{cre}(G) = 3$

Proof. The following are the three cases.

Case 1. If the set of vertices $S = \{u, v, w\}$ is independent, following that, there are three subcases.

Subcase 1. If $n = 3$, then $G \equiv N_3$, and $\gamma_{cre}(G) = 3$.

Subcase 2. If $n = 4, 5$, then the number of vertices that fall outside the set S is one or two in each case these vertices constitute the C^rEDS, since all vertices in the set S are adjacent to it. Thus, $\gamma_{cre}(G) = 1\ or\ 2$.

Subcase 3. If $n \geq 6$, then let $D = \{u, v, w\}$, every vertex in the set D is adjacent to every vertex in the set $V - D$ is. If not, in the set $V - D$, there is at least one vertex that is not next to any other vertex in the set D, say u. Thus, $deg(u) < n - 2$, since the set D is independent and this is a contradiction. Therefore, $\gamma_{cre}(G) = 3$.

Case 2. If $S = \{u, v, w\}$ is not independent,

Subcase 1. If $\langle S \rangle \equiv K_n$, then there are

I) If $n < n$, then there is no graph satisfied the conditions.

II) If $n = 5$, then, the set S's vertices are not next to the other two vertices. Thus, the graph $G \equiv K_3 \cup N_2$ or $K_3 \cup K_2$, so $\gamma_{cre}(G) = 3, 2$, respictively.

III) If $n = 6$, then let the set $V - S = \{v_1, v_2, v_3\}$, so there are

1) If the set $V - S$ is independent, so there are

a_1. If every vertex in the set S is next to a distinct vertex in the set $V - D$, then $G \equiv K_3 \odot K_1$, so $\gamma_{cre}(G) = 3$.

a_2. If two vertices of S say u and v are adjacent to the same vertex in the set $V - D$ say v_1 and the third vertex in the set S is next to another distinct vertex in the set $V - D$ say v_2. Then, the vertex v_3 is an isolated vertex. Let $D = \{v, w, v_3\}$, it is obvious that the set D is a DS and every vertex in the set $V - D$ is next to only one vertex in the set D. Then, D is C^1EDS and one can be concluded that it is MC^1EDS (as an example, see Figure 4.), so $\gamma_{cre}(G) = 3$.

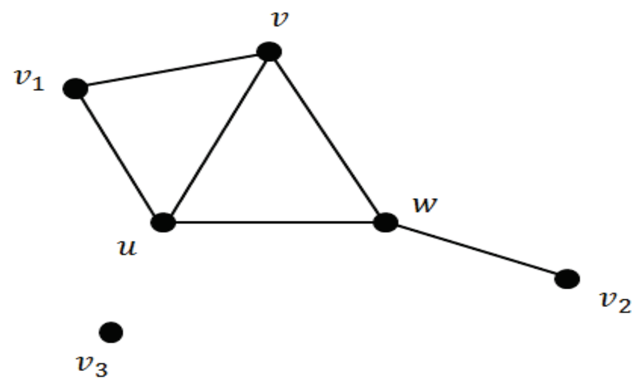


Figure 4

a_3. If all vertices of S are adjacent to the same vertex say v_1, then the two vertices v_2 and v_3 mean that $G \equiv K_4 \cup N_2$, then it is obvious that $\gamma_{cre}(G) = 3$.

2) 1) If the set $V - S \equiv K_2 \cup N_1$, K_3, or P_3, the in the same manner of (1), the same results is obtained.

If $n = 7$, so $V - S = \{v_1, v_2, v_3, v_4\}$ and the number of edges between the set s and the set is 6 to guaranty that each vertex in the set S has degree equal to $(n - 3 = 4)$.

Depending on the quantity of isolated vertices, there are three scenarios:

Case 1. If there are two isolated vertices say $\{v_3, v_4\}$, then every vertex in the set S is close to the other two vertices v_1 and v_2 together, so the degree of each one of these vertices is three (as an example, see Figure 5.). Let $D = \{v, v_3, v_4\}$, it is obvious that the set D is DS. Moreover, there is only one vertex in the set D that is next to every vertex in the set $V - D$ that is the vertex v. Therefore, the set D is C^1EDS, and it is evident that the set D is MC^1EDS, then $\gamma_{cre}(G) = 3$.

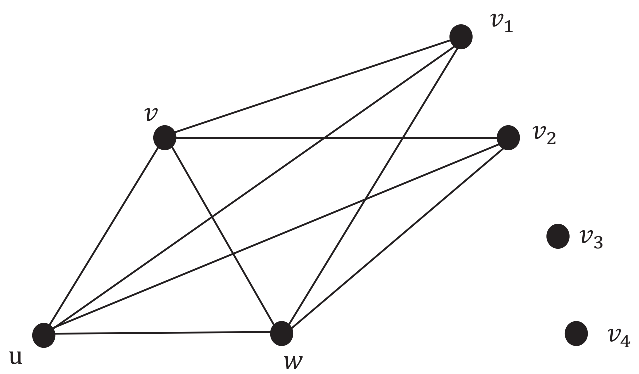
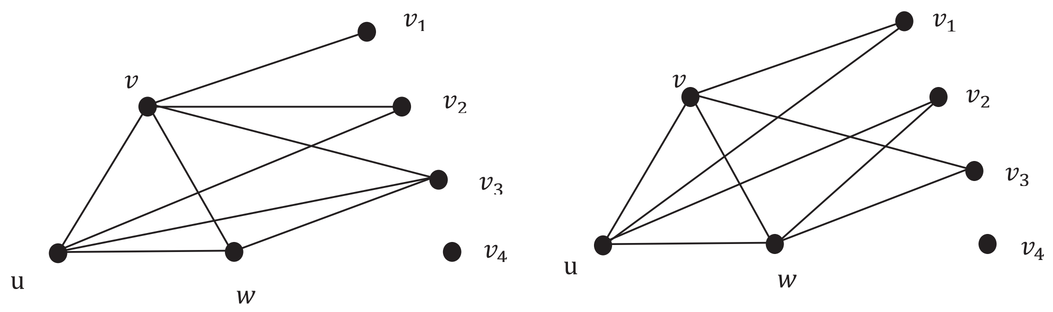


Figure 5. The MC^1EDS of order 7 with two isolated vertices.

Case 2. If there is one isolated vertices say $\{ v_4\}$, then there are two ways to join the six edges from the set S and the set V-S as follows.

I) The way $(1-2-3)$ that means one vertex of degree one say (v_1) and the the second of order two say (v_2) and the last (v_3) of order three (as an example, see Figure 6. (A)). Let $D = \{v_1, v_2, v_3, v_4\}$, then the set D is DS, and each vertex in V-D is adjacent to two vertices in the set D precisely. Additionally, the set D is MC^1EDS, then $\gamma_{cre}(G) = 4$.

II) The way $(2-2-2)$ that means $deg(v_i) = 2\ \forall i$ (as an example, see Figure 6. (b)). Let $D = \{v_1, v_2, v_3, v_4\}$, then the set D is DS, and each vertex in V-D is adjacent to two of the set D 's vertices exactly. Additionally, the set D is MC^1EDS, then $\gamma_{cre}(G) = 4$

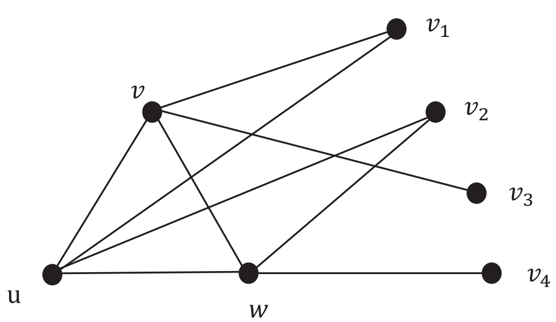


A B

Figure 6. The MC^1EDS of order 7 with one isolated vertices.

Case 3. If there is no isolated vertex, there are two ways to join the six edges from the set S and the set V-S as follows.
I) The way $(1-1-2-2)$ that means there are two vertices of order two and the other two vertices of order one (as an example, see Figure 7 (A)). Let $D = \{v_1, v_2, v_3, v_4\}$, then the set D is DS, and each vertex in V-D is adjacent to of the set D's vertices exactly. Additionally, set D is MC^1EDS, then $\gamma_{cre}(G) = 4$.

II) The way $(3-1-1-1)$ that means there is one vertex of order one and the other three vertices of order one (as an example, see Figure 7 (B)). In the same manner in in previous case (I), $\gamma_{cre}(G) = 4$.



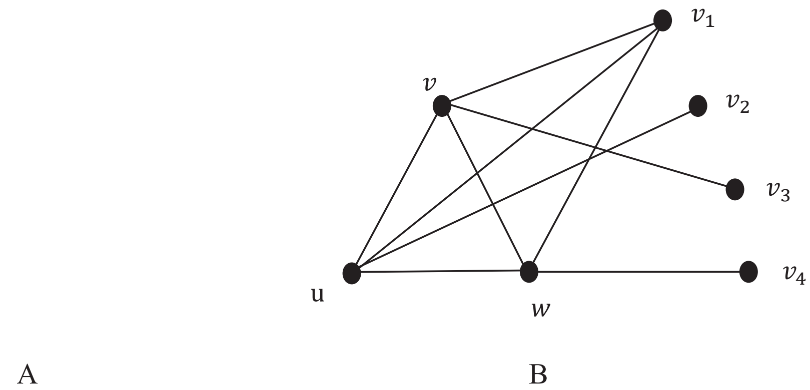


Figure 7. The MC^1EDS of order 7 with no isolated vertex.

3. Co r-Equality Domination of complement of some certain graphs

This section introduces the concept of Co r-Equality Domination of complement and provides proofs For some certain graphs.

Proposition 3.1. If G has Co r – equality of domination graph than $\bar{G}$ has an Co r- equality of domination.
Proof: Let G be a graph of order n , and if G has a vertex v such that $\deg(v) = n - 1$, then, v is isolated vertex in $\bar{G}$, then, it has co-r-equality domination number according to properties(2.3)

Theorem3.2. Given a path graph of order n, let $G = P_n$, $n \geq 2$, than, $\gamma_{cre}(\bar{G}) = 2$.
Proof. Let $G = P_n$, $n \geq 2$ then, the following three examples are dependent on n..
Case1. If $n = 2$. Thus, the $\overline{P_2}$ contains two isolated vertices then, $\gamma_{cre}(\overline{P_2}) = 2$ by properties 2.3.
Case 2. If $n = 3$, then, $\gamma_{cre}(\overline{P_2}) = 2$.
Case 3. If $n > 3$, thus, every vertex in P_n is nearby all vertices in $\overline{P_n}$ except the vertex which is next to it, according to the definition of complement. Let $D = \{v_1, v_n\}$ is a dominating set with the lowest cardinality in the graph $\overline{P_n}$ every vertex in D is near by $n - r$ vertices in $V - D$. Thus, the result is obtained. See figure 8

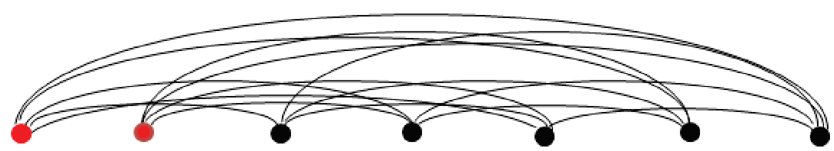


Figure 8.

Theorem3.3 . Let $G = C_n$ a cycle graph of order n, $n \geq 3$, than, $\gamma_{cre}(\bar{G}) = 2$.
Proof. Suppose $G = C_n$, $n \geq 3$ then, The following three examples are dependent on n
Case1. If $n = 3$. Then, the $\overline{C_3}$ contains three isolated vertices then, $\gamma_{cre}(\overline{C_3}) = 3$ by properties 2.3
Case 2. If $n = 4$, then, $\overline{C_4} = K_2 \cup K_2$, hence, let D be a set such that each vertex in K_2 includes exactly one vertex , so, it is simple to draw the conclusion that $\gamma_{cre}(\overline{C_4}) = 2$.
Case 3. If $n > 4$, then, according to the definition of

Complement graph, all vertices in $\overline{C_n}$ are next to every vertex in C_n, with the exception of the two vertices that are next to it.
. Let $D = \{ v_i , v_{i+1} \}$ is a dominating set it has the lowest cardinality in the graph $\overline{C_n}$ where each vertex in D is next to $n - r$ vertices in $V - D$. Then, the result is obtained.

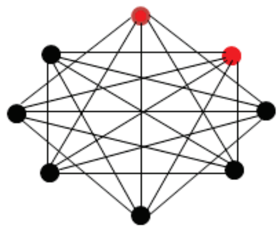


Figure 9.

Theorem3.4. . Assume that $G = k_n$ a complete graph of order n thus, $\gamma_{cre}(\overline{K_n}) = n$.
Proof. . Let $G = k_n$ a complete graph of order n then, $\overline{K_n}$ contains n isolated vertices by properties 2.3 $\gamma_{cre}(\overline{K_n}) = n$.
Theorem3.5. Assume that. $G = S_n$ a star graph of order n thus, $\gamma_{cre}(\overline{S_n}) = 2$.
Proof. Let $G = K_{1,n}$ such that $n \geq 1$, there are two cases depend on the order n .
Case 1. If $n = 2$ so, there are two isolated vertices by properties 2.3 $\gamma_{cre}(\overline{S_n}) = 2$.
Case 2. If $n > 2$ Afterwards , $\overline{S_n}$ contains dual component one of them isolated vertex and another in each component, a single vertex dominates all the others. Thus, $\gamma_{cre}(\overline{S_n}) = 2$.
Theorem3.6. Assuming that $G = W_n$ is a wheel graph of order n thus, $\gamma_{cre}(\overline{W_n}) = 3$.
Proof. Let $G = W_n$ a wheel graph of order $n \geq 3$, depending on the order n, there are two situations .
Case 1. If $n = 3$ thus, there are three isolated vertices by properties 2.3 $\gamma_{cre}(\overline{W_n}) = 3$.
Case 2. If $n > 3$ Afterwards , $\overline{W_n}$ contains dual component one of them isolated vertex and another in each component, there are only two vertices that predominate over the others. So, $\gamma_{cre}(\overline{W_n}) = 3$.
Theorem3.7. Let $G = K_{n,m}$ a complete bipartite graph of order $n, m \geq 2$ then, $\gamma_{cre}(\overline{K_{n,m}}) = 2$.
Proof. Let $\overline{K_{n,m}} \cong K_n \cup K_m$ depending on the two components' sequence, there are two situations. K_n and K_m .
Case 1. If $n = m$, then $\overline{K_{n,m}}$ has two components that are in the same order, meaning that each component's vertices are dominated by just one vertex thus, $\gamma_{cre}(\overline{K_{n,m}}) = 2$.
Case 2. If $n \neq m$, then $\overline{K_{n,m}}$ includes two parts that are arranged differently. At this point, assume without losing generality that $n < m$, therefore in the component K_n only one vertex has the ability to dominate over the remaining $n - 1$ vertices this vertex is part of the dominating set D contains this vertex. And the component K_m, $(m - (n - 1))$ the vertices must be added the set D. These vertices in the set D continue to dominate the same amount of the set's vertices $V - D$, then, $\gamma_{cre}(\overline{K_{n,m}}) = 2$.

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