Emad-israa transform a new integral transform of two parameters with applications

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Abstract. In the presented paper, authors introduced a new integral transform technique of two parameters “Emad –Israa Transform” with its fundamental properties and solved linear ordinary differential equations with constant coefficients. Numerical problems have taken for visualizing the complete procedure of solution. Results of these problems suggest that the present transform technique of two parameters “Emad- Israa” transform technique provides the outstanding results without dowing tedious computational work.

1 Introduction

Differential equations play an important role in our daily lives, explaining some of the laws of natural phenomena and working to solve their problems. They contribute fundamentally to finding solutions to some issues in various science fields, and among these fields are engineering with its branches, applied sciences, astronomy, solar energy generation, and others. Many papers and some books present the importance of integral transformations in solving problems of differential and integral equations of all kinds, problems of systems of ordinary and partial differential problems, problems of integro differential and their applications, obtaining exact solutions, and the benefit of converting these equations and systems into simple equations or algebraic systems. The most important of these applications are growth problems and issues of drug concentration, electrical circuits, image encryption, nuclear physics, space, astronomy, and others. [1-15].

Because of the importance of the differential equations mentioned above, one of the best ways to find the exact solution is to use integral transforms, which transform these difficult equations into simple algebraic equations that are easy to calculate. Many researchers have proposed many integral transforms, and each transform is not better than the other, but its importance lies according to the problem. The user who invited the suggestion [16,17,18,19,20,21,22]. The motive of the present paper is to develop a novel integral transform of two parameters, the "Emad-Israa" transform, with its fundamental properties, and solve linear ordinary differential problems with constant coefficients. The reason for choosing two different parameters for this integral transform

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is because of the importance of these two parameters in real life. Furthermore, the different powers in this integral transformation have two parameters.
The most important of the two parameter transforms are Formable transform [25], KKAT transform [26], Abaoub-Shkheam transform [23], Rishi transform [24], and others.

Emad- Israa Transform Technique

The proposed technique (transformation) is defined as function of an exponential order:

$$B = \{f(t): \exists K, m_1, m_2 > 0, |f(t)| < Ke^{m_j|t|} if\ t \in (-1)^j X[0,\infty), j = 1,2\} \quad (1)$$

Where: $f\ (t)$ is a function in B, K is a finite constant number, $m_1\ and\ m_2$ may or may not be finite.

Emad–Israa transform technique of two parameters symbolized by (EmIs) is defined by the integral equation:

$$EmIs\{\ f\ (t)\} = (rs)^n \int_0^\infty e^{-(rs)^m t} f(t)dt == \ T(r,s) \quad . \quad (2)$$

Where t > 0, m, n ∈ ℤ , $r, s\ > 0$ and r,s are two parameters that are applied as a factor to (t) in f.

In Emad– Israa technique (transformation), t in which is an argument of f is factored via the integral transform variable.

2 Applications of Emad- Israa Technique (Transformation) into Elementary Functions

In integral Emad- Israa technique, the existence condition must be met to use the transform for . $f(t)$

The existence condition to use *EmIs*$\{f(t)\}$ to $f(t)$:

For each t greater than or equal to(0) must be a piecewise continuous and in exponential order.

EmIs transform technique can be applied in finding the solution of the functions:

Assume that $f(t) = k$ is an arbitrary number , then

$$EmIs(k) = (rs)^n \int_0^\infty k\, e^{-(rs)^m t} dt = \frac{k(rs)^n}{(rs)^m} \quad , \ r \neq 0\ , s \neq 0.$$

let $f(t) = e^{at},\ then$

$$EmIs(e^{at}) = (rs)^n \int_0^\infty e^{at}\, e^{-(rs)^m t} dt = \frac{(rs)^n}{(rs)^m - a} \quad , (rs) > |a|.$$

let $f(t) = t^k \quad , \ k = 1,2,3, \ldots$

$$EmIs(t^k) = (rs)^n \int_0^\infty t^k\, e^{-(rs)^m t} dt = \frac{k!(rs)^n}{(rs)^{(k+1)m}} \quad , \ r \neq 0\ , \ s \neq 0.$$

Emad-Israa transform technique of two parameters on this function is important in evaluating sinusoidal functions.

$$EmIs[\sin at] = \frac{a(rs)^n}{(rs)^{2m} + a^2} \ , \ rs > a \quad , \quad EmIs[\cos at] = \frac{(rs)^{n+m}}{(rs)^{2m} + a^2} \ , rs > a.$$

$$EmIs[\sinh at] = \frac{a(rs)^n}{(rs)^{2m} - a^2} \ , rs > |a| \ , \ EmIs[\cosh at] = \frac{(rs)^n (rs)^m}{(rs)^{2m} - a^2} \ , rs > |a|.$$

$If\ EmIs(f(t)) = T(rs)\ \ then\ EmIs\{e^{-at} f(t)\} = T((rs) + a)$ (shifting property):

$$EmIs\{e^{-at} f(t)\} = (rs)^n \int_0^\infty e^{-at}\, f(t) e^{-(rs)^m t} dt \ ,$$

$$= (rs)^n \int_0^\infty f(t)\, e^{-[(rs)^m + a]t} dt = T\big((r,s) + a\big).$$

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$Let\ f(t) = t^k e^{-at}\ \ then:$

$$EmIs\{e^{-at}t^k\} = (rs)^n \int_0^\infty e^{-at}\, t^k e^{-(rs)^m t} dt \quad ,$$

$$= (rs)^n \int_0^\infty t^k\, e^{-[(rs)^m + a]t} dt \quad ,$$

$$= \frac{k!\,(rs)^n}{((rs)^m + a)^{(k+1)m}} \quad .$$

2.1 Theorem: let T(r,s)is Emad-Israa transform technique of [EmIs(f(t)=T(r,s)], then:

" Emad-Israa " technique of derivatives:
EmIs$[f'(t)] = -(rs)^n f(0) + (rs)^m T(r,s)$.

$$\text{EmIs}[f''(t)] = -(rs)^n f'(0) - (rs)^{m+n} f(0) + (rs)^{2m} T(r,s).$$

Proof:

$$EmIs[f'(t)] = (rs)^n \int_0^\infty e^{-(rs)^m t} f'(t) dt$$

Integration by parts to find that:

$$\text{EmIs}[f'(t)] = -(rs)^n f(0) + (rs)^m T(r,s)$$

$$Let\ g(t) = f'(t), then$$

$$\text{EmIs}[g'(t)] = -(rs)^n f'(0) + (rs)^m EmIs[f'(t)]$$

We find that by using (i),

$$\text{EmIs}[f''(t)] = -(rs)^n f'(0) - (rs)^{m+n} f(0) + (rs)^{2m} T(r,s).$$

3 Applications of Emad- Israa Transform technique into (O.D. E's):

For systems that governed via differential problems, Emad- Israa technique could be applied as an efficient tool to analyze their elementary characteristics in response into (I.C^s). initial conditions (data).

The efficiency of EmIs technique can be expressed in finding the exact solution of certain I.V.P. described via ordinary differential problems.

3.1 Applications of EmIs transform technique into 1^{st}ordinary differential problems

Consider the 1^{st}ordinary differential problem:

$\frac{dy}{dt} + py = f(t), t > 0$ with (I.C.) initial data: y (0) :=a

Where a , p are arbitrary numbers and f(t) is an external input function f so that EmIs(f (t)) technique exists.

By using Emad-Israa technique on the upper problem:

$$-(rs)^n f(0) + (rs)^m T(r,s) + pT(r,s) = \bar{f}(r,s)$$

$$T(r,s)(p + (rs)^m) = \bar{f}(r,s) + a(rs)^n$$

$$T(r,s) = \frac{\bar{f}(r,s) + a(rs)^n}{(p + (rs)^m)} \ .$$

It is possible to evaluate f(t) via applying EmIs inverse technique.

3.2 Applications of EmIs transform technique of second-order ordinary differential equations

For a 2^{nd} order linear differential problem that has the general formula: $\frac{d^2y}{dt^2}+2p\frac{dy}{dt}+qy=f(t), t>0$, the initial data: y(0)= a ,$y'(0)=b$ a,b,p , q are arbitrary numbers.

Using Emad- Israa technique of f (t) with *I.V. Ps* a, b, p , q would get:

$$T(r,s)=\frac{f(t)+b(rs)^n}{((rs)^{2m}+2p(rs)^m+q)}+\frac{a(rs)^{n+m}}{((rs)^{2m}+2p(rs)^m+q)}+\frac{2ap(rs)^n}{((rs)^{2m}+2p(rs)^m+q)},$$

it is possible to evaluate f (t) via applying EmIs inverse technique.

4 Examples: To increase the comprehension of integral Emad-Israa technique, useful examples would be represented:

Example(A): To find the exact solution of the 1^{st}differential problem :$\frac{dy}{dt}-y=0$,$y(0)=1$.

Applying EmIs technique:

$-(rs)^n y(0)+(rs)^m T(r,s)-T(r,s)=0 \to T(r,s)((rs)^m-1)=(rs)^n$

$$T(r,s)=\frac{(rs)^n}{((rs)^m-1)}$$

EmIs inverse technique would be:$y(t)=e^t$ (exact solution)

Example(B): To solve the 1^{st} order differential problem:

$y''+y=t$,$y(0)=1$,$y'(0)=2$.

EmIs technique for the eq. is:

$-(rs)^n y'(0)-(rs)^{m+n}y(0)+(rs)^{2m}T(r,s)+T(r,s)=\frac{(rs)^n}{(rs)^{2m}}$,

$-2(rs)^n-(rs)^{m+n}+T(r,s)((rs)^{2m}+1)=\frac{(rs)^n}{(rs)^{2m}}$,

Then, $T(r,s)=\frac{(rs)^n}{(rs)^{2m}[(rs)^{2m}+1]}+\frac{2(rs)^n+(rs)^m(rs)^n}{[(rs)^{2m}+1]}$.

Now let $\frac{(rs)^n+2(rs)^n(rs)^{2m}+(rs)^{3m}(rs)^n}{(rs)^{2m}[(rs)^{2m}+1]}=\frac{A}{(rs)^m}+\frac{B}{(rs)^{2m}}+\frac{C(rs)^m+D}{[(rs)^{2m}+1]}$.

After basic computations we have:

A=0, B=$(rs)^n$, C=$(rs)^n$, D=$(rs)^n$.

Then

$$T(r,s)=\frac{(rs)^n}{(rs)^{2m}}+\frac{(rs)^n(rs)^m}{[(rs)^{2m}+1]}+\frac{(rs)^n}{[(rs)^{2m}+1]} .$$

The inverse transform of above equation gives the exact solution $y(t)=t+\cos t+\sin t$

Example (C): To find the exact solution of the following differential problem: $y''-3y'+2y=4e^{2t}$,$y(0)=-3$,$y'(0)=5$.

EmIs transform technique for the eq. is:

$-(rs)^n y'(0)-(rs)^{m+n}y(0)+(rs)^{2m}T(r,s)+3(rs)^n y(0)-3(rs)^m T(r,s)+2T(r,s)=4\frac{(rs)^n}{(rs)^m-2}$,

$$-5(rs)^n+3(rs)^{m+n}+(rs)^{2m}T(r,s)-9(rs)^n-3(rs)^m T(r,s)+2T(r,s)=\frac{4(rs)^n}{(rs)^m-2},$$

$$-14(rs)^n + 3(rs)^{n+m} + T(r,s)[(rs)^{2m} - 3(rs)^m + 2] = \frac{4(rs)^n}{(rs)^m - 2} \quad ,$$

$$T(r,s)[((rs)^m - 2)((rs)^m - 1)] = \frac{4(rs)^n}{(rs)^m - 2} + 14(rs)^n - 3(rs)^{n+m} \quad ,$$

$$T(r,s) = \frac{-3(rs)^{2m+n} + 20(rs)^{m+n} - 24(rs)^n}{[(rs)^m - 1][(rs)^m - 2]^2} \quad ,$$

$$\frac{-3(rs)^{2m+n} + 20(rs)^{m+n} - 24(rs)^n}{[(rs)^m - 1][(rs)^m - 2]^2} = \frac{A}{[(rs)^m - 1]} + \frac{B}{[(rs)^m - 2]} + \frac{C}{[(rs)^m - 2]^2} \; .$$

We have (by fractional part)

$A = -7(rs)^n, B = 4(rs)^n, C = 4(rs)^n$.

Then, $T(r,s) = \frac{-7(rs)^n}{[(rs)^m-1]} + \frac{4(rs)^n}{[(rs)^m-2]} + \frac{4(rs)^n}{[(rs)^m-2]^2}$.

EmIs inverse technique would be:

$$y(t) = -7e^t + 4e^{2t} + 4te^{2t} \quad .$$

Example (D): Applying Emad- Israa technique to find the solution of Law Newton of application

Based on (law of Newton of cooling), an object surrounded via an environment with the temperature is different than the object it changes appropriately to the difference between the temperature of object and its surroundings.

Assume that:

ω is the body temperature at time τ,

ω_0 is the surrounding environment temperature,

Therefore, the law of Newton of cooling relation would be:

$\frac{d\omega}{d\tau} \alpha[\omega(\tau) - \omega_0(\tau)] \Rightarrow \frac{d\omega}{d\tau} = -\mu[\omega(\tau) - \omega_0(\tau)]$, (1)

Where μ is a constant proportionality ($\mu > 0$).

The problem: if temperature of a body is drops from eighty degrees Celsius to sixty degrees Celsius in twenty min., when placed in air, and surrounding air temperature is forty degrees Celsius.

What will be temperature of a body after forty min.?

When will temperature of a body reach fifty-five degrees Celsius?

Solution:

Consider the following:

The surrounding air temperature is: $\omega_0 =$ forty degrees Celsius,

Initial temperature of body is ω (0) = 80°C at the time $\tau = 0$,

After time $\tau =$ twenty min., the temperature of body becomes ω(twenty) = sixty degrees Celsius .

Using Newton's law of cooling in eq. (1):

$$\frac{d\omega}{d\tau} \alpha[\omega(\tau) - \omega_0(\tau)] \, ,$$

$$\Rightarrow \frac{d\omega}{d\tau} = -\mu[\omega(\tau) - \omega_0(\tau)] \, , where\ \mu\ is\ a\ constant,$$

$$\Rightarrow \frac{d\omega}{d\tau} = -\mu\, \omega(\tau) + \mu\, \omega_0(\tau) \, ,$$

$\Rightarrow \frac{d\omega}{d\tau} + \mu\, \omega(\tau) = \mu\omega_0(\tau)$ $or\ \omega'(\tau) + \mu\omega(\tau) = 40\omega,$ (2)

Taking Emad-Israa technique to both sides of eq. (2) obtains:

$EmIs\{\omega'(\tau)\} + \mu\, EmIs\{\omega(\tau)\} = 40\mu EmIs\{1\}.$

Applying Emad-Israa technique of derivatives theorem, gets:

$-(rs)^n\omega(0) + (rs)^m T(r,s) + \mu T(r,s) = \frac{40\mu(rs)^n}{(rs)^m}$ (3)

Since it is given that $\omega(0) = 80°\ at\ the\ time\ \ \tau = 0,$
$then\ substituting\ \omega(0)\ \ into\ eq.(3)\ gives$:

$$-80(rs)^n + T(r,s)[(rs)^m + \mu] = \frac{40\mu(rs)^n}{(rs)^m}$$

$$T(r,s)[(rs)^m + \mu] = \frac{40\mu(rs)^n}{(rs)^m} + 80(rs)^n$$

$$T(r,s)[(rs)^m + \beta] = \frac{40\mu(rs)^n + 80(rs)^n(rs)^m}{(rs)^m[(rs)^m + \mu]}$$

$$\frac{40\mu(rs)^n + 80(rs)^n(rs)^m}{(rs)^m[(rs)^m + \mu]} = \frac{A}{(rs)^m} + \frac{B}{[(rs)^m + \mu]}$$

We obtain (by fractional part)
$A = 40(rs)^n \quad , B = 40(rs)^n$

$Then,\ T(r,s) = \frac{40(rs)^n}{(rs)^m} + \frac{40(rs)^n}{[(rs)^m+\mu]}$, (4)

EmIs inverse technique would be:
$\omega(\tau) = 40 + 40e^{-\mu\tau}$, (5)

Since it is given that $\omega(20) = 60°C\ \ at\ \tau = 20\ \ then$:
$60 = 40 + 40e^{-20\mu} \ \Rightarrow\ 20 = 40e^{-20\mu}$,

$\frac{1}{2} = e^{-20\mu} \ \Rightarrow \frac{1}{2} = (e^{-\mu})^{20} \Rightarrow \left(\frac{1}{2}\right)^{\frac{1}{20}} = e^{-\mu}$, (6)

It is possible to the temperature of body at time $\tau = forty$ applying eq. (6) as:

$$\omega(\tau) = 40 + 40e^{-40\mu}$$

$$\omega(\tau) = 40 + 40(e^{-\mu})^{40} \Rightarrow \omega(\tau) = 40 + 40\left(\frac{1}{2}\right)^{\frac{40}{20}},$$

$$\omega(\tau) = 40 + 40\left(\frac{1}{2}\right)^{20}$$

$$\omega(\tau) = 50℃.$$

So, after time $\tau = 40$ min., the temperature of body is equal to $fifty$℃
To evaluate the required time τ for the temperature of body to reach $55°C$:

$$55 = 40 + 40e^{-\mu\tau} \ \Rightarrow 15 = 40(e^{-\mu})^{\tau},$$

$\frac{15}{40} = (e^{-\mu})^{\tau}$. (7)

Substituting eq. (6) in eq. (7), gives:

$$\frac{15}{40} = \left(\frac{1}{2}\right)^{\frac{1}{20}}.$$

Using the natural logarithm (Ln) to both sides of the result gets:

$$\ln\left(\frac{15}{40}\right) = \frac{1}{20}\ln\left(\frac{1}{2}\right) \Rightarrow \tau = \frac{20\ \ln\left(\frac{15}{40}\right)}{\ln\left(\frac{1}{2}\right)}.$$

$\therefore \tau = 28.3$ min.
So, the reaches of the temperature of body after 28.3 min.

Example(E): Applying Emad-Israa technique to find the solution of the (A Uniformly Loaded Beam Problem)

The fourth order linear differential problem with boundary conditions (data) that represent the beam deflection under a uniform load of α_0 per unit with hinges $= 0$, $\tau = l$ at its ends is:

$$\frac{d^4\mathrm{X}}{d\tau^4} = \frac{\alpha_0}{EI} \quad , \qquad 0 < \tau < 1 \tag{8}$$

Such that:

$E \ \equiv Young's\ modulus,$
I
$\equiv the\ cross$
$-\ section's\ moment\ of\ inertia\ about\ an\ axis\ normal\ to\ the\ plane\ of\ bending,$
$EI \ \equiv the\ beam's\ flexural\ rigidity,$
$\mathrm{X}'(0) \equiv the\ beam's\ slop,$
$\mathrm{B}(\tau) = EI\mathrm{X}''(t) is\ the\ beam's\ bending\ moment,$
$S(\tau) = M'(\tau) = EI\mathrm{X}''(t) is\ the\ beam's\ shear\ at\ a\ point.$

It is possible to find the exact solution of deflection of the uniformly loaded beam applying Emad-Israa technique through the application of the technique to both sides of eq. (8):

$$-(rs)^n\mathrm{X}'''(0) - (rs)^{m+n}\mathrm{X}''(0) - (rs)^{2m+n}\mathrm{X}'(0) - (rs)^{3m+n}\mathrm{X}(0) + (rs)^{4m}T(r,s) = \frac{\varphi_0\ (rs)^n}{EI\ (rs)^m} \tag{9}$$

Applying the 1^{st} and 2^{nd} conditions and the unknown conditions(data):

$y'(0) = C_1\ \ and\ y'''(0) = C_2\ into\ equation\ (9), obtains$:

$$-C_2(rs)^n - C_1(rs)^{2m+n} + (rs)^{4m}T(r,s) = \frac{\varphi_0\ (rs)^n}{EI\ (rs)^m}\ ,$$

$$(rs)^{4m}T(r,s) = \frac{\varphi_0\ (rs)^n}{EI\ (rs)^m} + C_2(rs)^n + C_1(rs)^{2m+n}$$

$$T(r,s) = \frac{\varphi_0\ (rs)^n + C_2(rs)^{m+n} + C_1(rs)^{3m+n}}{EI\ (rs)^{5m}}$$

$$T(r,s) = \frac{\varphi_0\ (rs)^n}{EI\ (rs)^{5m}} + \frac{C_2(rs)^n}{(rs)^{4m}} + \frac{C_1(rs)^n}{(rs)^{2m}}$$

Applying the inverse Emad-Israa technique to find the exact solution:

$$\mathrm{X}(t) = \frac{\varphi_0}{4\ EI}\tau^5 + \frac{C_2}{3}\tau^4 + C_1\tau.$$

Conclusion

We conclude from the paper that there is no integral transformation that is better than the other transformation. Each transformation is important according to the problem for which the exact solution is to be found, and that the two parameters in this transformation were useful in facilitating the algebraic calculations.

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