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Another kind of general integral transform called "emad-salam-elaf transform"

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Abstract. This article introduces another type of integral transformation in its general format; this general transformation is proposed as a new general integral transformation called the "Emad-Salam-Elaf Transform." This general integral transformation is elaborated, and its fundamental functions and theorems have been proven. The eligibility of the proposed general integral transform is proven by solving ordinary differential equations of the first and second-order nature.Another type of general integral transform has been studied and proven for some elementary functions, its applicability and ability to evaluate the exact solution have been proven by utilizing the transform to find the solution of first and second order ordinary differential equations.

1 Introduction

Integral transforms are of great importance in finding the exact solution to differential equations and are characterized by their simplicity and ease (Abbas et al. 2022, Maktoof et al. 2021, Mansour et al. 2021a, Meddahi et al. 2022).
Many researchers, starting with Laplace, have developed general formulas for integral transforms, (Mansour et al. 2022, Maktoof et al. 2021, Meddahi et al. 2022, Filipinas 2017). These transforms also have great importance in finding exact solutions to physical, engineering, medical, and nuclear problems, and even in astronomy and economics. There is much papers into these applications, (Kuffi et al. 2020, Mansour et al. 2021b, Mansour et al. 2021c, Saadeh et al. 2022, Waqas et al. 2022, Moazzam et al. 2023).

In (2017), (Filipinas 2017) presented an integral transform called "Rangaig Transform". The formula of Rangaig transform is:

$$\eta\{f(t)\} = A(\mu) = \frac{1}{\mu}\int_{-\infty}^{0} e^{\mu t} f(t)dt.$$

Also, in (2022), (Mansour et al. 2021a), an integral transform called Generalized of Rangaig integral transform is introduced. The general Rangaig transform equation is:

$$\eta_g\{f(t)\} = A_g(\mu) = \frac{1}{\mu^n}\int_{-\infty}^{0} e^{\mu t} f(t)dt\,, n \in \mathbb{Z}\,.$$

2 ANOTHER KIND OF GENERAL INTEGRAL TRANSFORM

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The prospect of the proposed general transformation is not any different from other integral transforms, which are concentrated on transforming an equation from one domain into another while maintaining the exact solution of that equation. It is possible to define the exponential order of a set of functions as:

$$G_g = \{f(t): \exists\, N, \lambda_1 \; and \; \lambda_2 > 0 \,, |f(t)| > Ne^{\lambda j|t|}\}$$
$$wher \; t \in (-1)^{j-1} \times (-\infty, 0)\,, j = 1,2 \qquad (1)$$

For Eq. (1):

N is a finite constant.

$\lambda_1 \; and \; \lambda_2$ are constants that are finite or infinite.

The general integral transform that is defined in Eq.(1) for the set G_g is:

$$EA\{f(t)\} = R(\mu) = g(\mu) \int_{t=-\infty}^{0} e^{p(\mu)t} \; f(t)dt \qquad (2)$$

Where $R(\mu)$ represents the general integral transform of $f(t) \in G_g$.

$\frac{1}{\lambda} < \mu < \frac{1}{\lambda_2}$. $p(\mu)$ and $g(\mu)$ are functions of parameter μ. For the function $f(t)$, t is factorized by the parameter μ, or the function $f(t)$ is a mapped into the function $R(\mu)$ of μ- space. This integral transform was named "Emad-Salam-Elaf transform".

3 THE ANOTHER KIND OF GENERAL INTEGRAL TRANSFORM FOR SOME BASIC FUNCTIONS

For the function $f(t)$ that is existed and an element of G_g, another kind of general integral transform is a picewise continuous and has decreasing exponential order if $t \le 0$ is obtained for the function $f(t)$.

Theorem (A)

If $f(t) = k$, where k is a constant, then its general integral transform is :

$$EA\{k\} = g(\mu) \int_{-\infty}^{0} e^{p(\mu)t} k \; dt = \frac{kg(\mu)}{p(\mu)}.$$

And, if $f(t) = t$, then its general integral transform is

$$EA\{t\} = g(\mu) \int_{-\infty}^{0} e^{p(\mu)t} t \; dk = \frac{-g(\mu)}{[p(\mu)]^2}.$$

Now, the general integral transform for $f(t) = t^n$, $n \in \mathbb{N}$ is :

$$EA\{t^n\} = g(\mu) \int_{-\infty}^{0} e^{p(\mu)t} \, t^n dt = \frac{(-1)^n g(\mu) n!}{[p(\mu)]^{n+1}}.$$

Proof: From general integral transform definition:

$$EA\{t^n\} = g(\mu) \int_{-\infty}^{0} e^{p(\mu)t} \, t^n dt = g(\mu) \left[\frac{t^n \, e^{p(\mu)t}}{p(\mu)}\right]_{-\infty}^{0} - \frac{n}{p(\mu)} \int_{-\infty}^{0} e^{p(\mu)t} \, t^{n-1} dt \;\; .$$
$$= [\frac{g(\mu)}{p(\mu)} \, e^{p(\mu)t} \left(t^n - \frac{n \, t^{n-1}}{p(\mu)} + \frac{n(n-1) \, t^{n-2}}{[p(\mu)]^2} - \cdots + \frac{n! \, (-1)^n}{[p(\mu)]^n}\right]_{-\infty}^{0}$$
$$= \frac{n! \, (-1)^n \, g(\mu)}{[p(\mu)]^{n+1}} \; .$$

Theorem (B)

For an exponential function $f(t) = e^{at}$, where a is a constant, then its general integral transform is

$$EA\{e^{at}\} = g(\mu) \int_{-\infty}^{0} e^{p(\mu)t} \, e^{at} \, dt = \frac{g(\mu)}{p(u) + a} \, , \qquad p(\mu) > a.$$

Theorem (C)

The general integral transform for trigonometric functions which are $f(t) = \sin(t)$ and $f(t) = \cos(t)$, are:

$$EA\{\sin(t)\} = \frac{-g(\mu)}{[p(\mu)]^2 + 1},$$

and

$$EA\{\cos(t)\} = \frac{g(\mu)\, p(\mu)}{[p(\mu)]^2 + 1}.$$

Proof: The exponential format trigonometric functions can be written as:

$sin(t) = \frac{1}{2i}(e^{it} - e^{-it}), i = \sqrt{-1}$, then

$$EA\{\sin(t)\} = g(\mu) \int_{-\infty}^{0} e^{p(\mu)t} \sin(t)\, dt = \frac{g(\mu)}{2i} \int_{-\infty}^{0} e^{p(\mu)t}\, (e^{it} - e^{-it}) dt,$$

$$= \frac{g(\mu)}{2i} \int_{-\infty}^{0} e^{(p(\mu)+i)t}\, dt - \frac{g(\mu)}{2i} \int_{-\infty}^{0} e^{-(i-p(\mu)t}\, dt,$$

$$= \frac{g(\mu)}{2i} \left[\frac{1}{p(\mu) + i} + \frac{1}{i - p(\mu)}\right] = \frac{g(\mu)}{2i} \left[\frac{i - p(\mu) + i + p(\mu)}{(p(\mu) + i)(i - p(\mu))}\right],$$

$$= g(\mu) \left[\frac{-1}{[p(\mu)]^2 + 1}\right] = \frac{-g(\mu)}{[p(\mu)]^2 + 1}.$$

Similarly, for $\cos(t) = \frac{1}{2}(e^{it} + e^{-it}),\ i = \sqrt{-1}$, then

$$EA\{\cos(t)\} = \frac{g(\mu)}{2} \int_{-\infty}^{0} e^{p(\mu)t}\, (e^{it} + e^{-it}) dt = \frac{g(\mu)}{2} \int_{-\infty}^{0} e^{(p(\mu)+i)t}\, dt + \frac{g(\mu)}{2} \int_{-\infty}^{0} e^{-(i-p(\mu))t}\, dt,$$

$$= \frac{g(\mu)}{2} \left[\frac{1}{p(\mu) + i} - \frac{1}{i - p(\mu)}\right] \mu = \frac{g(\mu)}{2} \left[\frac{i - p(\mu) - i - p(\mu)}{ip(\mu) - [p(\mu)]^2 - 1 - ip(\mu)}\right]$$

$$= \frac{g(\mu) p(\mu)}{[p(\mu)]^2 + 1}.$$

Theorem (D): (The general integral transform for donatives)

If $f(t), f'(t), f''(t), \dots, f^{(n)}(t) \in G_g, n \in \mathbb{N}$, then

$EA\{f'(t)\} = g(\mu)\, f(0) - p(\mu)\, EA\{f(t)\}$.

(ii) $EA\{f''(t)\} = g(\mu)\, f'(0) - g(\mu)\, p(\mu) f(0) + [p(\mu)]^2\, EA\{f(t)\}$.

Proof:

(i) $EA\{f'(t)\} = g(\mu) \int_{-\infty}^{0} e^{p(\mu)t}\, f'(t)\, dt$.

Integrating by parts,

Let $u = e^{p(\mu)}, dv = f'(t)\, dt,$

$du = p(\mu) e^{p(\mu)t}\, dt, v = f(t).$

$EA\{f(t)\} = g(\mu) \left[f(t)\, e^{p(\mu)t}\right|_{-\infty}^{0} - p(\mu) \int_{-\infty}^{0} f(t)\ e^{p(\mu)t}\, dt]$

$= g(\mu)\, f(0) - p(\mu)\, EA\{f(t)\}.$

(ii) Similarly, integrating by parts

Let $u = e^{p(\mu)t}$, $dv = f''(t) dt$

$du = p(\mu) e^{p(\mu)t}$, $v = f'(t).$

$EA\{f''(t)\} = g(\mu) \left[e^{p(\mu)} f'(t) |_{-\infty}^{0} - p(\mu) \int_{-\infty}^{0} f'(t)\ e^{p(\mu)t}\, dt\right],$

$= g(\mu) f'(0) - p(\mu)\, EA\{f'(t)\},$

$= g(\mu)f'(0) - p(\mu)[g(\mu)f(0) - p(\mu)EA\{f(t)\}],$
$= g(\mu)f'(0) - g(\mu)\, P(\mu)f(0) + [p(\mu)]^2 EA\{f(t)\}.$

In general:

$$\mathrm{EA}\{f^{(n)}(t)\} = g(\mu) \sum_{j=0}^{n-1} (-1)^j\, [p(\mu)]^j\, f^{(n-1-j)}(0) + (-1)^n [p(\mu)]^n\, EA\{f(t)\}\,.$$

4 APPLICATIONS OF ANOTHER KIND OF GENERAL INTEGRAL TRANSFORM "EMAD- SALAM TRANSFORM"

Examples of solving first and second ordinary differential equations are given to expound the implementable of the introduced general integral transformation.

Application (A)

The relation representing radioactive decay is, [2]

$$N'(t) = -\lambda\, N(t)\; with\; N(0) = N_0\,. \tag{3}$$

$N(t)$ is the number of undecayed atoms left in a sample of radioactive isotope, and λ is a decay constant. The radioactive material decay rate is proportional to the amount present after three hours for an initial amount of (100) milligrams of the radioactive material. It is a 20% reduction in the radioactive material's original mass, (Kuffi et al. 2020). The half-life of radioactive substance can be estimated applying the general integral transform. By using the general integral transform to both sides of Eq. (3), we get:

$EA\{N'(t)\} = -\lambda\, EA\{N(t)\},$
$g(\mu)\, N(0) - p(\mu)\, EA\,\{N(t)\} + \lambda\, EA\,\{N(t)\} = 0,$
$g(\mu)\, N_0 - (p(\mu) - \lambda)EA\,\{N(t)\} = 0,$
$-(p(u) - \lambda)EA\{N(t)\} = -N_0 g(\mu),$
$EA\,\{N(t)\} = \dfrac{N_0 g(\mu)}{p(\mu) - \lambda}\,.$

Taking the inverse of the general integral transform gives:

$$N(t) = N_0\, e^{-\lambda t}. \tag{4}$$

Applying the condition when $t = 0$, then $N_0 = 100$ to Eq. (4), then:

$$N(t) = 100\, e^{-\lambda t}. \tag{5}$$

At $t = 3$ then $N(t) = 100 - 20 = 80$ and so Eq. (5) becomes: $80 = 100e^{-3\lambda}$,

So,

$$\lambda = -\frac{1}{3}\ln\left(\frac{4}{5}\right)\,. \tag{6}$$

Since the time require when $N(t) = \frac{N_0}{2} = 50$, we get:

$$50 = 100\, e^{-\lambda t}\,. \tag{7}$$

Putting Eq. (6) into Eq. (7), we get:

$$50 = 100\, e^{[\frac{t}{3}\ln(\frac{4}{5})]}\,. \tag{8}$$

Simplifying for t from Eq. (8), obtains: $t = 9.32$ hours.

Application (B)

The 2^{nd} order linear ordinary differential equation:

$\frac{d^2y}{dt^2} + y(t) = 1,$ with $y'(0) = 0$ and $y(0) = 0,$ it is possible to apply general integral transform to find the solution of the above equation as:

By taking the general integral transform for both sides of the above equation:

$$EA\{y''\} + EA\{y\} = EA\{1\},$$

$$g(\mu)y'(0) - g(\mu)p(\mu)\, y(0) + [p(\mu)]^2 EA\{y(t)\} + EA\{y(t)\} = \frac{-g(\mu)}{[p(\mu)]^2}\,.$$

Then:

$$EA\{y(t)\} = \frac{-g(\mu)}{[p(\mu)]^2([P(\mu)]^2 + 1))} \ .$$

After simple computation and taking the invers of the general integral transform, then the exact solution in obtained:

$$y(t) = 1 - \cos(t).$$

5 CONCLUSIONS

Another type of the general integral transform, the Emad-Salam-Elaf transform, is a derived transform from the well-known Rangaig integral transform and generalized of the Rangaig transform that provides a general integral transform; the characteristic of the proposed transformation have been examined by using the transform on some rudimentary functions. Applying this transform in solving ordinary differential equations of the first and second order has displayed the qualification and efficiency of the proposed transform to be employed in such essential equations applied in many scientific fields.

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