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Dual soft condensed sets

Janat Y. Rasheed[1*] and *L. A. A.* Jabar[1]

[1]Department of Mathematics, College of Education for Pure Sciences, University of Babylon, Babylon, Iraq.

Abstract The work here was based on some topological concepts and studied within dual soft set theory where three types of condensed were identified within these sets and their topological effects were highlighted.

1 Introduction

The scientific and life problems that Human faces in different times and eras are considered the main pillar of growth process in all mathematical sciences and through them the rest of the sciences grow and flourish. Here the researchers put themselves in front of difficult and important challenges to build and modify some mathematical concepts in a way that some mathematical concepts in a way that is consistent with facilitating on finding appropriate solutions to these problems. One of the most important concepts that gained brilliance and wide distinction is what Zadeh introduced in 1965 [1] which are the fuzzy sets that invaded all science wit out exception and played an important role in the process of growth and development that the human world is witnessing in 1999 soft sets appeared ,which were compatible with the path of fuzzy sets ,being define in $X \times 2^E$space [2,3] while fuzzy sets were define in $X \times [0,1]$ spaces .Soft sets invaded a wide field in various sciences ,then center sets were identified in proximity spaces by Abdulsada and Al-Swidi in 2019 [4]. During this year (2022) Luay et al. [5-7] build dual soft sets under space $X \times 2^{E1} \times 2^{E2}$where E_1 and E_2 are parameters for X elements.

2 Preliminaries

2.1 Definition 2.1: [8]

Let U_1 and U_2 are initial universes sets and E be the set of all possible parameters under consideration with respect to U_1 and U_2 .Whereas parameters are description features on properties of members of the initials universes sets. The triple (A, F, G) is called dual soft set over to $U_1$and U_2

where F, G are functions where $F : A \to P\,(U_1)$, $G : A \to P(U_2)$. i.e., $A_{FG} = \{(e, F(e), G, (e)\ \forall\ e\ \in A\ , (e, \emptyset, \emptyset)\ \forall\ e\ \in E/A)\}$. The collection of all dual soft sets denoted by $Ds\ (U_1, U_2)_E$

* Corresponding author : janat.rasheed.pure393@student.uobabylon.edu.iq

2.2 Definition 2.2: [8]

The dual empty soft set $\emptyset_{Ds} = \{(e, \emptyset, \emptyset) ; \forall\, e \in E\}$.
The dual absolute soft set $X_{Ds} = \{(e, U1, U2);\ \forall\, e \in E\}$.
The singular dual soft point $SD_e\ at\ e \in E$; $SD_e = \big(e, F(e), G, (e)\big)$.
The dual soft point DP_e at e is dual soft point such that $DP_e\ (\acute{e}) = \big(e, F(e), G, (e)\big)$ for $\acute{e}\ = e$ and is equal to $(e, \emptyset, \emptyset)$ if $\acute{e}\ \neq e$.

2.3 Definition 2.3: [8]

The dual soft union of two dual soft sets A_{F1G1} and B_{F2G2} over a common universes sets U_1 and U_2 with A, B are subsets of parameters E over a member for U_1 and U_2 is defined as the dual soft set (c, H_1, H_2) where $C = A \cup B$ and $\big(c, H_1(c), H_2(c)\big) = \big(c, F_1(c) \cup F_2(c), G_1(c) \cup G_2(c)\big)\ if\ c \in A \cup B$ and $= (c, \emptyset, \emptyset)\ if\ c \in\ E\,/C$ denoted by $A_{F1G1}\ \ \cup_{DS}\ B_{F2G2}$

2.4 Definition 2.4: [8]

The dual soft intersection of two dual soft sets A_{F1G1} and B_{F2G2} with A, B are subsets of parameters E of members for U_1 and U_2 is defined as dual soft set (c, H_1, H_2) where $C = A \cup B$ and $\big(c, H_1(c), H_2(c)\big) = \big(c, F_2(c) \cap F_2(c), G_1(c) \cap G_2(c)\big)\ if\ c \in A \cup B$ and $= (c, \emptyset, \emptyset)\ if\ c \in\ E\,/C$ denoted by $A_{F1G1}\ \ \cap_{DS}\ \ B_{F2G2}$.

2.5 Definition 2.5: [8]

The complement of dual soft set $(A, F, G)$is denoted by $(A, F, G)^e = A\ \ F^e\ \ G^e$
Where
$F^e : E \ \rightarrow P(U_1) \quad G^e : E \ \rightarrow P(U_2)$
$F^e(e)\ = U_1 -\ F\ (e) \quad G^e(e) = U_2 - G(e)\ \ \forall e \in E$

2.6 Theorem 2.6: [8] (The Demorgan's law)

The triple $(\cap_{DS}, \cup_{DS}$, complement) is dual soft sets.

2.7 Definition 2.7: [8]

The pair (X_{Ds}, T_E) is called dual soft topological space if T_E satisfies, first $\emptyset_{Ds}, X_{Ds}\ \in\ X_{Ds}$ second T_E is closed under finite dual soft intersection and finally closed under dual soft union of any sub collection of T_E .

Now we will discuss the concept of dual soft interior, dual soft closure and some of their properties:

2.8 Definition 2.8

Let $(X_{Ds}, T_E\ )$ be a dual soft topological space and let $A_{FG} \subseteq X_{Ds}$ the dual soft interior point $DS_i\ (A_{FG}) = \cup_{e\,\in\,E}\ \{DP_e\ \ \in X_{Ds}\ ;\ \exists\, U_{F1G1}\ \in\ T_E\ \ni\ DP_e\ \in\ U_{F1G1} \subseteq A_{FG}\}$.

2.9 Theorem 2.9

Let (X_{Ds}, T_E) be dual soft topological space and let $A_{FG}, B_{F1G1} \subseteq X_{Ds}$ then:

$DS_i(X_{Ds}) = X_{Ds}, DS_i(\emptyset_{Ds}) = \emptyset_{Ds}$.

$DS_i(A_{FG}) \subseteq A_{FG}$.

If $A_{FG} \subseteq B_{F1G1}$ then $DS_i(A_{FG}) \subseteq DS_i(B_{F1G1})$.

$DS_i(A_{FG} \cup_{DS} B_{F1G1}) \subseteq DS_i(A_{FG}) \cup_{DS} DS_i(B_{F1G1})$.

$DS_i(A_{FG} \cap_{DS} B_{F1G1}) = DS_i(A_{FG}) \cap_{DS} DS_i(B_{F1G1})$.

$DS_i[DS_i(A_{FG})] = DS_i(A_{FG})$.

2.10 Definition 2.10

Let (X_{Ds}, T_E) be a dual soft topological space and let $A_{FG} \subseteq X_{Ds}$ the dual soft closure of A_{FG} is $DS_{cl}(A_{FG}) = \cup_{e\in E}\{DP_e \in X_{Ds}, \forall U_{F1G1} \in T_E \ni U_{F1G1} \cap_{DS} A_{FG} \neq \emptyset_{Ds}\}$.

2.11 Theorem 2.11

Let(X_{Ds}, T_E) be dual soft topological space and let $A_{FG}, B_{F1G1} \subseteq X_{Ds}$ then:

$DS_{cl}(A_{FG}) = \cap \{A_{1F1G1}, A_{1F1G1}$is dual soft closed set and $A_{FG} \subseteq A_{1F1G1}\}$

$DS_{cl}(\emptyset_{Ds}) = \emptyset_{Ds}, DS_{cl}(X_{Ds}) = X_{Ds}$.

If $A_{FG} \subseteq B_{F1G1}$ then $DS_{cl}(A_{FG}) \subseteq DS_{cl}(B_{F1G1})$.

$A_{FG} \subseteq DS_{cl}(A_{FG})$.

$DS_{cl}(A_{FG} \cup_{DS} B_{F1G1}) = DS_{cl}(A_{FG}) \cup_{DS} DS_{cl}(B_{F1G1})$.

$DS_{cl}(A_{FG} \cap_{DS} B_{F1G1}) \subseteq DS_{cl}(A_{FG}) \cap_{DS} DS_{cl}(B_{F1G1})$.

$DS_{cl}[DS_{cl}(A_{FG})] = DS_{cl}(A_{FG})$.

The following proposition characteristic that link the dual soft interior point with dual soft closure point and these characteristics are consider pillar on which the research path depends.

2.12 Proposition 2.12

For any dual soft set $A_{FG} \subseteq X_{Ds}$ the following relation hold

If $U_{F1G1} \in T_E$ Then $U_{F1G1} \cap_{DS} DS_{cl}(A_{FG}) \subseteq DS_{cl}(U_{F1G1} \cap_{DS} A_{FG})$ for any $A_{FG} \in X_{DS}$.

If A_{FG} or $B_{F2G2} \in T_E$ then $DS_i[(DS_{cl} A_{FG} \cap_{DS} B_{F2G2})] = DS_i[DS_{cl}(A_{FG})] \cap_{DS} DS_i[DS_{cl}(B_{F2G2})]$.

$DS_i[DS_{cl}(DS_i(DS_{cl}(A_{FG})))] = DS_i[DS_{cl}(A_{FG})]$.

$DS_{cl}[DS_i(DS_{cl}(DS_i(A_{FG})))] = DS_{cl}[DS_i(A_{FG})]$.

$DS_i[DS_{cl}(DS\ A_{FG})_i)] \cap_{DS} DS_i[DS_{cl}(DS_i(B_{F2G2}))] = DS_i[DS_{cl}(DS_i(A_{FG} \cap_{DS} B_{F2G2}))]$.

$DS_{cl}[DS_i(DS_{cl}(A_{FG}))] \cup_{DS} DS_{cl}[DS_i(DS_{cl}(B_{F2G2}))] = DS_{cl}[DS_i(DS_{cl}(A_{FG} \cup_{DS} B_{F2G2}))]$.

2.12.1 Proof

Let DP_e be any dual soft point $DP_e \in U_{F1G1} \cap_{DS} DS_{cl}(A_{FG})$

Then

$$\forall V_{F3G3} \in T_E(DP_e) \ni V_{FiGi} \cap_{DS} A_{FG} \neq \emptyset_{Ds} \quad (2.1)$$

If possible $DP_e \notin DS_{cl}(U_{F1G1} \cap_{DS} A_{FG})$ so $\exists\ W_{F4G4} \in T_E(DP_e) \ni W_{F4G4} \cap_{DS} U_{F1G1} \cup_{DS} A_{FG} = \emptyset_{Ds}$ but $W_{F4G4} \cap_{DS} U_{F1G1} \in T_E(DP_e)$ which contradict to (2.1)

Hence $DP_e \in DS_{cl}(U_{F1G1} \cap_{DS} A_{FG})$

Now we will introduce a new concept that represents the main focus of our research:

2.13 Definition 2.13

A dual soft set A_{FG} of X_{DS} is said to be

Dual soft super condensed if set DS_i $[DS_{cl}$ $(A_{FG})] = DS_i$ (A_{FG})

Dual soft semi condensed set if DS_{cl} $[DS_i(A_{FG})$ $] =$ DS_{cl}

Dual soft condensed if set DS_i $[DS_{cl}$ $(A_{FG})]$=DS_i (A_{FG}) and DS_{cl} $[DS_i$ $(A_{FG}))]$=DS_{cl} (A_{FG})

2.14 Example 2.14

Let U_1= $\{h_1 , h_2 , h_3\}$ U_2= $\{c_1 , c_2, c_3\}$ E= $\{e_1 , e_2\}$

A_{F1G1} = $\{(e_1, \{h_1\}, \{c_1\}), (e_2 , \{h_2\}, \{c_2\})\}$

B_{F2G2} = $\{(e_1 , \{h_2 , h_3\}, \{ c_2 , c_3\}) , (e_2, \{h_1, h_3\}, \{c_1 , c_3\})\}$

T_E = $\{\emptyset_{DS} , X_{DS}, A_{F1G1}, B_{F2G2}\}$

DS_{cl} $(DS_i$ $(A_{F1G1}$ $)) = A_{F1G1}$

$(DS_{cl}$ $(A_{F1G1})) = A_{F1G1}$ therefore A_{F1G1} is dual soft condensed set.

From above definition we can conclude that any dual soft closed set is dual soft super condensed. Also, any dual soft open set is dual soft semi condensed. Finally, a dual soft set A_{FG} is dual soft condensed set if and only if it is dual soft super condensed set and dual soft semi condensed set.

The question that arises is what is the relationship between the dual soft condensed sets and theirs complement that is will be answered in the next theorem.

2.15 Theorem 2.15

Let $(X_{Ds} , T_E$ $)$ be a dual soft topological space then the following statements are holds

The dual soft complement of a dual soft super condensed set is dual soft semi condensed set

The dual soft complement of a dual soft semi condensed set is dual soft super condensed set

The dual soft complement of dual soft condensed set is also dual soft condensed set.

2.15.1 Proof

Let A_{F1G1} be a dual soft super condensed set

then we have $X_{Ds} - [DS_i(DS_{cl}$ $(A_{F1G1}))] =$ $X_{Ds} - [DS_i(A_{F1G1})]$

Considering the properties of dual soft complement, we get

X_{Ds}– $[$ $DS_i(DS_{cl}(A_{F1G1}))]$=$DS_{cl}[X_{Ds}$ $-(DS_{cl}$ $(A_{F1G1}))]$= $DS_{cl}[DS_i(X_{Ds} - A_{F1G1})]$

$X_{Ds} - [DS_i$ $(A_{F1G1})] = DS_{cl}$ $(X_{Ds} - A_{F1G1})$ then $DS_{cl}[DS_i(X_{Ds} - A_{F1G1})]$ =DS_{cl} $(X_{Ds}$ $- A_{F1G1})$ So X_{Ds} $-$ A_{F1G1} is dual soft semi condensed set.

There are some properties in addition the concept of dual soft condensed, super condensed and semi condensed:

2.16 Theorem 2.16

Let $(X_{Ds} , T_E$ $)$ be a dual soft topological space then the following statement are hold

A dual soft set $A_{F1G1}$is dual soft super condensed set if and only if DS_i $[DS_{cl}$ $(A_{F1G1})] \subseteq A_{F1G1}$.

A dual soft set B_{F2G2} is dual soft semi condensed set if and only if DS_{cl} $[DS_i(B_{F2G2})] \supseteq B_{F2G2}$.

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A dual soft set A_{F1G1} of X_{Ds} is dual soft condensed set if and only.
If $DS_i\ [DS_{cl}\ (A_{F1G1})] \subseteq A_{F1G1} \subseteq DS_{cl}\ [DS_i(\ A_{F1G1})]$.

2.16.1 Proof

1-Let A_{F1G1} be a dual soft super condensed set then we have $DS_i[DS_{cl}(A_{F1G1})] = DS_i$ $(A_{F1G1}) \subseteq A_{F1G1}$

conversely $DS_i\ [\ DS_{cl}\ (A_{F1G1})] \subseteq A_{F1G1}$ taking dual soft interior for both side

$DS_i\ [DS_{cl}\ (A_{F1G1})] \subseteq DS_i\ (A_{F1G1})$

considering the self- evident relation$DS_i\ [DS_{cl}\ (A_{F1G1})] \supseteq DS_i\ (A_{F1G1})$

So $DS_i\ [DS_{cl}\ (A_{F1G1})] = DS_i\ (A_{F1G1})$

Therefore A_{F1G1} is dual soft super condensed set.

2-Let B_{F2G2} is dual soft semi condensed set then we have

$DS_{cl}[DS_i\ (B_{F2G2})]= DS_{cl}\ (B_{F2G2}) \supseteq_{DS} B_{F2G2}$

Conversely let we assume $B_{F2G2} \subseteq_{DS} DS_{cl}(DS_i\ (B_{F2G2}))$

Taking dual soft closure for both side we have

$DS_{cl}\ (B_{F2G2}\ ) \subseteq_{DS} DS_{cl}[DS_i\ (B_{F2G2})]$

Considering the self - evident relation $DS_{cl}\ (DS_i\ (B_{F2G2}\ )) \subseteq_{DS} DS_{cl}\ (B_{F2G2})$

We obtain $DS_{cl}\ (B_{F2G2}) = DS_{cl}[DS_i\ (B_{F2G2})]$

Then $B_{F2G2}$is dual soft semi condensed set

3-Directly by definition 2.13

2.17 Definition 2.17

A dual soft set A_{FG} of X_{Ds} is called

Dual soft regular open set if $DS_i\ [DS_{cl}\ (A_{FG}\ )] = A_{FG}$.

Dual soft regular closed set if $DS_{cl}\ [DS_i\ (A_{FG})] = A_{FG}$.

by Example 2.14 we find $DS_i\ [DS_{cl}(A_{F1G1})] = A_{F1G1}$

Then A_{F1G1} is dual soft regular open set

$DS_{cl}\ [\ DS_i\ (A_{F1G1})\ ] = A_{F1G1}$

Then A_{F1G1} is dual soft regular closed set.

From the previous definition we can drive some conclusions: that is the dual soft whole space X_{Ds} and the dual soft empty set $\emptyset_{Ds}$ are dual soft regular open set and also dual soft regular closed set. furthermore the dual soft complement of dual soft regular open set is dual soft regular closed set, the dual soft complement of dual soft regular closed set is dual soft regular open set, also the dual soft inter section of two dual soft regular open set is dual soft open set, the dual soft union of two dual soft regular closed set is dual soft regular closed set, finally for any dual soft set A_{FG} then $DS_i\ [DS_{cl}\ (A_{FG})]$ is dual soft regular open set, $DS_{cl}\ [DS_i$ $(A_{FG})]$ is dual soft regular closed set .

From above argument we set that the collection of all dual regular open set building a base for topology of X_{Ds}.

As it known that the dual soft boundary of A_{FG} which denoted by $DS_{fr}\ (A_{FG}) = DS_{cl}$ $(A_{FG}) - DS_i\ (A_{FG}) = DS_{cl}\ (A_{FG}) \cap_{DS} DS_{cl}\ [X_{Ds} - (A_{FG})]$

If we want to clarify some of the relationships between the dual soft regular open or closed sets and the dual soft condensed, super condensed, semi condensed sets we can use the following theorem:

2.18 Theorem 2.18

For any dual soft set $A_{FG} \subseteq X_{Ds}$The following relation hold

A dual soft set is dual soft regular open set if and only if the dual soft set is dual soft super condensed set.

A dual soft set is dual soft regular closed if and only if the dual soft set is dual soft semi condensed set.

A dual soft set is dual soft regular open set if and only if the dual soft set is dual soft condensed and open set.

A dual soft set is dual soft regular closed set if and only if the dual soft set is dual soft condensed and closed set.

A dual soft set A_{FG} is dual soft condensed set if and only if there is a dual soft regular open set U_{F1G1} such that $U_{F1G1} \subseteq A_{FG} \subseteq DS_{cl}(U_{F1G1})$.

A dual soft set B_{F2G2} is dual soft condensed set if and only if there is a dual soft regular closed set V_{F3G3} such that $DS_i(V_{F3G3}) \subseteq B_{F2G2} \subseteq V_{F3G3}$

2.18.1 Proof

1-Let A_{FG} be a dual soft regular open set then

$$DS_i[DS_{cl}(A_{FG})] = A_{FG} \quad (2.2)$$

So $DS_i(A_{FG}) = DS_i[DS_i(DS_{cl}(A_{FG}))] = DS_i[DS_{cl}(A_{FG})] = A_{FG}$

this implies that A_{FG} is dual soft open set.

From Eq. (2.2) we get

$DS_i[DS_{cl}(A_{FG})] = DS_i(A_{FG})$

Thus A_{FG} is dual soft super condensed set.

Conversely let A_{FG} be dual soft super condensed and open set then we have $DS_i[DS_{cl}(A_{FG})] = DS_i(A_{FG}) = A_{FG}$. Thence A_{FG} is dual soft regular open set.

2- Let A_{FG} be a dual soft regular closed set

Then $DS_{cl}[DS_i(A_{FG})] = A_{FG}$ ----------- (2)

Using this relation we get $DS_{cl}(A_{FG}) = DS_{cl}[DS_{cl}(DS_i(A_{FG}))] = DS_{cl}[DS_i(A_{FG})] = A_{FG}$

This implies that A_{FG} is dual soft closed set

From equation (2) we get $DS_{cl}[DS_i(A_{FG})] = DS_{cl}(A_{FG})$

Then A_{FG} is dual soft semi condensed set

Conversely let A_{FG} be a dual soft semi condensed and closed set then we have

$DS_{cl}[DS_i(A_{FG})] = DS_{cl}(A_{FG}) = A_{FG}$

Thus A_{FG} is dual soft regular closed set

(3) and (4) directly

5-Let A_{FG} is dual soft condensed set if we take $U_{F1G1} = DS_i[DS_{cl}(A_{FG})]$ is dual soft open set by remark 2.13 then $U_{F1G1} = DS_i[DS_{cl}(A_{FG})] = DS_i(A_{FG}) \subseteq_{DS} A_{FG}$

since A_{FG} is dual soft condensed set then $U_{F1G1} \subseteq_{DS} A_{FG}$

And $DS_{cl}(U_{F1G1}) = DS_{cl}(DS_i(DS_{cl}(A_{FG}))) \supseteq_{DS} DS_i[DS_{cl}(A_{FG})] DS_{cl}(A_{FG}) \supseteq_{DS} A_{FG}$

So U_{F1G1} satisfies $U_{F1G1} \subseteq_{DS} A_{FG} \subseteq_{DS} DS_{cl}(U_{F1G1})$

If we assume that there is a dual soft regular open set U_{F1G1} which is satisfies $U_{F1G1} \subseteq_{DS} A_{FG} \subseteq_{DS} DS_{cl}(U_{F1G1})$ then

$DS_i[DS_{cl}(A_{FG})] \subseteq DS_i[DS_{cl}(DS_{cl}(U_{F1G1}))] = DS_i[DS_{cl}(U_{F1G1})] = U_{F1G1}$

$DS_i[DS_{cl}(A_{FG})] \supseteq_{DS} DS_i[DS_{cl} U_{F1G1})] = U_{F1G1}$

So $U_{F1G1} = DS_i[DS_{cl}(A_{FG})]$. On other hand

$DS_{cl}[DS_i(A_{FG})] \subseteq_{DS} DS_{cl}[DS_i(DS_{cl}(U_{F1G1}))] = DS_{cl}(U_{F1G1})$

$DS_{cl}[DS_i(A_{FG})] \supseteq_{DS} DS_{cl}[DS_i(U_{F1G1})] = DS_{cl}(U_{F1G1})$

So $DS_{cl}(U_{F1G1}) = DS_{cl}[DS_i(A_{FG})]$ Then we have $DS_i[DS_{cl}(A_{FG})] \subseteq_{DS} A_{FG} \subseteq_{DS} DS_{cl}[DS_i(A_{FG})]$.So $DS_i[DS_i(DS_{cl}(A_{FG}))] = DS_i[DS_{cl}(A_{FG})] \subseteq_{DS} DS_i(A_{FG})$.But $DS_i(A_{FG}) \subseteq_{DS} DS_{cl}[DS_i(A_{FG})]$.Thus $DS_i(A_{FG}) = DS_{cl}[DS_i(A_{FG})]$

Also $A_{FG} \subseteq_{DS} DS_{cl}[DS_i\ (A_{FG})]$ so $DS_{cl}(A_{FG}) \subseteq_{DS} DS_{cl}[DS_i\ (A_{FG})]$.But $DS_{cl}[DS_i\ (A_{FG})] \subseteq_{DS} DS_{cl}(A_{FG})$.Therefore A_{FG} is dual soft condensed set.

6 - Assume B_{F2G2} is dual soft condensed set and let we take

$V_{F3G3} = DS_{cl}[DS_i\ (B_{F2G2})]$ then $V_{F3G3}$is dual soft regular closed set

Because $V_{F3G3}= DS_{cl}[DS_i\ (B_{F2G2})] = DS_{cl}\ (B_{F2G2}) \supseteq_{DS} B_{F2G2}$

And $DS_i(V_{F3G3})= DS_i[DS_{cl}(DS_i\ (B_{F2G2}))] \subseteq_{DS} DS_i[DS_{cl}\ (B_{F2G2})]= DS_i(B_{F2G2}) \subseteq_{DS} B_{F2G2}$.So V_{F3G3} satisfies $DS_i\ (V_{F3G3}) \subseteq_{DS} B_{F2G2} \subseteq_{DS} V_{F3G3}$. If we assume that there is a dual soft regular closed set V_{F3G3} which satisfies $DS_i(V_{F3G3}) \subseteq_{DS} B_{F2G2} \subseteq_{DS} V_{F3G3}$ .Then $V_{F3G3} = DS_{cl}[DS_i\ (V_{F3G3})] = DS_{cl}[DS_i\ (DS_i\ (V_{F3G3}))] \subseteq_{DS} DS_{cl}\ [DS_i\ (B_{F2G2})]$

$V_{F3G3} = DS_{cl}[DS_i\ (V_{F3G3})] \supseteq_{DS} DS_{cl}[DS_i\ (B_{F2G2})]$So $V_{F3G3} = DS_{cl}[DS_i\ (B_{F2G2})]$ On other hand$DS_i[DS_{cl}\ (B_{F2G2})] \supseteq DS_i[DS_{cl}\ (DS_i\ (V_{F3G3}))]= DS_i\ (V_{F3G3})= DS_i[DS_{cl}\ (B_{F2G2})] \subseteq_{DS}$

$DS_i\ [DS_{cl}\ (V_{F3G3})] = DS_i\ (V_{F3G3}\ )$ .So $DS_i\ (V_{F3G3}) = DS_i[DS_{cl}\ (B_{F2G2})]$ Then we have $DS_{cl}[DS_i\ (B_{F2G2})] \supseteq_{DS} B_{F2G2} \supseteq_{DS} DS_i[DS_{cl}\ (B_{F2G2})]$

So B_{F2G2} is dual soft condensed

$DS_{cl}[DS_i\ (B_{F2G2})]= DS_{cl}\ (B_{F2G2}\ )$

$DS_i[DS_{cl}\ (B_{F2G2})] = DS_i\ (B_{F2G2}\ )$.

The question that comes to our mind is how to link by dual soft condensed sets with the dual soft border which we will try to explain through the following theorem

2.19 Theorem 2.19

A dual soft set A_{FG} is dual soft condensed set if and only if the dual soft boundary of A_{FG} coincides with

$DS_{cl}\ [DS_i\ (A_{FG})] - DS_i\ [DS_{cl}\ (A_{FG})]= DS_{cl}[DS_i\ (A_{FG})] \cap_{DS} DS_{cl}\ [DS_i\ (X_{Ds} - (A_{FG}))]$

2.19.1 Proof

We assume that A_{FG} is a dual soft condensed set

by Theorem 2.15 part (3) $X_{Ds} - A_{FG}$ is dual soft condensed set.

So, we have $DS_{cl}(DS_i\ (A_{FG})) = DS_{cl}(A_{FG})$

$DS_{cl}[DS_i\ (X_{Ds} - A_{FG})] = DS_{cl}(X_{Ds} - A_{FG})$

Therefor $DS_{cl}[DS_i(A_{FG})] \cap_{DS} DS_{cl}[DS_i(X_{Ds} - A_{FG})]$

$= DS_{cl}\ (A_{FG}) \cap_{DS} DS_{cl}\ (X_{Ds} - A_{FG}) = DS_{fr}(A_{FG}\ )$

Now $DS_{fr}\ (A_{FG}\ )= DS_{cl}[DS_i(A_{FG}\ )] \cap_{DS} DS_{cl}[\ DS_i\ (\ X_{Ds} - A_{FG})]$

Then $DS_{cl}(A_{FG})= DS_i(A_{FG}) \cup_{DS} DS_{fr}\ (A_{FG}\ )$

$= DS_i\ (A_{FG}) \cup_{DS} \{\ DS_{cl}[\ DS_i\ (A_{FG})] \cap_{DS} DS_{cl}[DS_i\ (X_{Ds} - A_{FG})]\}$

$\subseteq DS_i\ (A_{FG}) \cup_{DS} DS_{cl}[DS_i\ (A_{FG})] = DS_{cl}[DS_i\ (A_{FG})]$

Considering the evident relation $DS_{cl}(A_{FG}) \supseteq DS_{cl}[\ DS_i\ (A_{FG})]$

$$DS_{cl}\ (A_{FG}) = DS_{cl}\ [DS_i\ (A_{FG})] \quad (2.3)$$

The same relation holds for $X_{Ds} - A_{FG}$because the dual soft boundary of $X_{Ds} - A_{FG}$ coincides with that of A_{FG} so we have

$DS_{cl}\ (X_{Ds} - A_{FG}) = DS_{cl}[DS_i\ (X_{Ds} - A_{FG})]$

$DS_{cl}\ (X_{Ds} - A_{FG}\ ) = X_{Ds} - [DS_i\ (A_{FG})]$

$DS_{cl}\ [DS_i\ (X_{Ds} - A_{FG})] = DS_{cl}[X_{Ds} - DS_{cl}\ (A_{FG})] = X_{Ds} - [DS_i\ (DS_{cl}\ (A_{FG}))]$

Thus $X_{Ds} - [DS_i\ (A_{FG})] = X_{Ds} - [DS_{cl}\ (DS_i\ (A_{FG}))]$

Taking dual soft complement for both side we obtain

$DS_i\ (A_{FG}) = DS_i\ (DS_{cl}\ (A_{FG}))$

This equation and Eq. (2.3) imply that the dual soft set A_{FG} is dual soft condensed set. Now we will give a mathematical concept that has a direct impact on the concept that we presented in Definition 2.17 and the basic role is the dual soft boundary points:

2.20 Definition 2.20

The dual soft interior of the dual soft boundary of the dual soft set is called dual soft border of the dual soft set by Example 2.14 we can find DS_{fr} $(A_{F1G1}) = DS_{cl}$ $(A_{F1G1}) \cap_{DS} DS_{cl}$ $[X_{Ds} - (A_{F1G1})] = \emptyset_{DS}$

now DS_i $[DS_{fr}$ $(A_{F1G1})] = \emptyset_{DS}$Note that the dual soft border of any dual soft set A_{FG} is denoted by DS_b $(A_{FG})$is expressed by DS_i $[DS_{fr}$ $(A_{FG})]$.

2.21 Theorem 2.21

The dual soft border of any dual soft set A_{FG} is dual soft regular open and expressed by

DS_b $(A_{FG}) = DS_i$ $[DS_{cl}$ $(A_{FG})] - DS_{cl}$ $[DS_i$ $(A_{FG})] = DS_i$ $[DS_{cl}$ $(A_{FG})] \cap_{DS} DS_i$ $[DS_{cl}$ $(X_{Ds} - (A_{FG}))]$

2.21.1 Proof

Considering to the fact that the dual soft boundary of A_{FG} is dual soft closed set then we have

DS_b $(A_{FG}) = DS_i$ $[DS_{fr}$ $(A_{FG})] = DS_i$ $[DS_{cl}$ $(DS_{fr}$ $(A_{FG}))]$

Since DS_i $[DS_{cl}$ $(A_{FG})]$ is dual soft regular open set DS_b (A_{FG}) we conclude that DS_b (A_{FG}) is dual soft regular open set.

Now we will prove the second part of the theorem

DS_i $[DS_{cl}$ $(A_{FG})] - DS_{cl}$ $[DS_i$ $(A_{FG})] = DS_i[DS_{cl}$ $(A_{FG})] \cap_{DS} DS_i$ $[DS_{cl}$ $(X_{Ds} - (A_{FG}))] = DS_i$ $[DS_{cl}$ $(A_{FG})] \cap_{DS} DS_i$ $[DS_{cl}$ $(X_{Ds} - (A_{FG}))]$

$= DS_i$ $[DS_{cl}$ $(A_{FG}) \cap_{DS} DS_{cl}$ $(X_{Ds} - (A_{FG}))]$ by property of dual soft interior

$= DS_i$ $[DS_{fr}$ $(A_{FG})]$ (Definition of dual soft boundary)

$= DS_b$ (A_{FG}) (Definition of dual soft border)

2.22 Theorem 2.22

For any dual soft set $A_{FG} \subseteq X_{Ds}$ then

A dual soft set is dual soft super condensed set if and only if

The dual soft interior of the set is dual soft regular open set and

The dual soft border of the dual soft set is empty

A dual soft set is dual soft semi condensed if and only if

The dual soft closure of the dual soft set is dual soft regular closed and

The dual soft border of the dual soft set is empty

A dual soft set is dual soft condensed if and only if

The dual soft interior of the dual soft set is dual soft regular open

The dual soft closure of the dual soft set is dual soft regular closed set

The dual soft border of the dual soft set is empty

DS_b $(DS_b$ $(A_{FG})) = \emptyset_{DS}$

2.22.1 Proof

Let A_{FG} is dual soft super condensed set then

$$DS_i[DS_{cl}(A_{FG})] = DS_i(A_{FG}) \quad (2.4)$$

So we get

$DS_i[(DS_{cl}(DS_i\ (DS_{cl}\ (A_{FG})))] = DS_i[(DS_{cl}DS_i(A_{FG}))]$

So the left side of the last equality become

$DS_i[DS_{cl}(DS_i\ (DS_{cl}\ (A_{FG})))] = DS_i[(DS_{cl}A_{FG})]$ by Proposition 2.12 part 3

$= DS_i\ (A_{FG})$ by Eq. (2.4)

Subsequently $DS_i\ (A_{FG}) = DS_i\ [DS_{cl}\ (DS_i\ (A_{FG}))]$

This implies that $DS_i\ (A_{FG})$ is dual soft regular open set using Eq. (2.4) we get

$DS_i\ [DS_{cl}(A_{FG})] = DS_i\ (A_{FG}) \subseteq DS_{cl}[DS_i\ (A_{FG})]$

$DS_b\ (A_{FG}) = DS_i\ [DS_{cl}\ (A_{FG})] - DS_{cl}\ [DS_i\ (A_{FG})] = \emptyset_{DS}$

Conversely suppose that A_{FG} satisfies that

$$DS_i[DS_{cl}\ (DS_i\ (A_{FG})] = DS_i\ (A_{FG}) \quad (2.5)$$

And $DS_b\ (A_{FG}) = \emptyset_{DS}$

$DS_i[DS_{cl}\ (A_{FG})] \subseteq DS_{cl}[DS_i\ (A_{FG})]$

Taking dual soft interior for both side of the second relation we get

$DS_i\ [DS_{cl}\ (A_{FG})] \subseteq DS_i(DS_{cl}(DS_i\ (A_{FG})))$

Combining this relation with (2.5) we get $DS_i\ (A_{FG}) \supseteq DS_i\ [DS_{cl}\ (A_{FG})]$

Considering the self - evident relation $DS_i\ (A_{FG}\ ) \subseteq DS_i[DS_{cl}(A_{FG})]$

we get $DS_i\ (A_{FG}) = DS_i[DS_{cl}\ (A_{FG})]$.

This relation implies that A_{FG} is dual soft super condensed set.

Through what has been studied we can classify the dual soft sets by this definition to the following classification:

2.23 Definition 2.23

Any dual soft set of X_{Ds} can be classified into one of the following three types

$Su(X_{DS})\{A_{FG}: DS_i\ [DS_{cl}\ (A_{FG})] \subset DS_{cl}\ [DS_i\ (A_{FG})], A_{FG} \subseteq X_{Ds}\}$

$Se(X_{DS})\{B_{FG} : DS_i\ [DS_{cl}\ (B_{FG})] \supsetneq DS_{cl}\ [DS_i\ (B_{FG})], B_{FG} \subseteq X_{Ds}\}$

$So(X_{DS})\{C_{FG}: DS_i\ [DS_{cl}(C_{FG})], DS_{cl}[DS_i\ (C_{FG}\ )]$ are non- comparable, $C_{FG} \subseteq X_{Ds}\}$

2.24 Example 2.24

Let $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2, c_3\}$, $E = \{e_1, e_2, e_3\}$

$A_{1F1G1} = \{(e_1\ , \{h_1\}, \{c_1\}), (e_2, \{h_3\}, \{c_3\})\}$

$A_{2F2G2} = \{(e_1\ , \emptyset\ , \{\ c_3\})\ , (e_2\ , \{\ h_2\}, \emptyset)\}$

$A_{3F3G3} = \{(e_1\ , \{h_1\}, \{c_1\ , c_3\}), (e_2\ , \{h_2\ , h_3\}, \{c_3\})\}$

$T_E = \{\emptyset_{DS}\ , X_{Ds}, A_{1F1G1}\ , A_{2F2G2}\ , A_{3F3G3}\ \}$

$X_{Ds} - A_{1F1G1} = \{(e_1, \{h_2\ , h_3\}, \{c_2\ , c_3\}), (e_2\ , \{h_1\ , h_2\}\ , \{c_1\ , c_2\})\}$

$X_{Ds} - A_{2F2G2} = \{\ (e_1\ , U_1\ , \{c_1\ , c_2\}), (e_2\ , \{h_1\ , h_3\}, U_2)\}$

$X_{Ds} - A_{3F3G3} = \{\ (e_1\ , \{\ h_2\ , h_3\ \}\ , \{\ c_2\ \})\ , (e_2\ , \{h_1\}, \{c_1\ , c_2\})\}$

Assume $A_{FG} = \{(e_1, \{\ h_1\ , h_2\}, \{\ c_1\ , c_2\}), (e_2, \{\ h_1\ , h_3\}, \{\ c_2\ , c_3\})\}$

Now $DS_{cl}[DS_i\ (A_{FG})] = DS_{cl}\ (A_{1F1G1}) = X_{Ds} - A_{2F2G2}$

$DS_i[DS_{cl}(A_{FG})] = DS_i\ (X_{Ds} - A_{2F2G2}) = A_{1F1G1} \subseteq_{DS} X_{Ds} - A_{2F2G2}$

We see $DS_i(DS_{cl}\ (A_{FG})) \subseteq_{DS} DS_{cl}\ (DS_i\ (A_{FG}))$.

Therefor A_{FG} belong to $Su(X_{DS})$.

If we compare the previous definition with what we learned about the concept of dual soft condensed, super condensed and semi condensed sets we will find that the dual soft set belong to the $Su(X_{DS})$ if and only if the dual soft border of the set is empty so a dual soft super condensed set and a dual soft semi condensed set is belong to $Su(X_{DS})$.

Again the question arise can we classify the complements of these types of dual soft sets as the original sets and that is what will be do in the following theorem:

2.25 Theorem 2.25

For any dual soft set A_{FG}belong to $Su(X_{DS})$ or $Se(X_{DS})$ or $So(X_{DS})$ then $X_{Ds} - A_{FG}$ belong to $Su(X_{DS})$ or $Se(X_{DS})$or $So(X_{DS})$ respectively

2.25.1 Proof

If we assume that A_{FG} belong to $Su(X_{DS})$

Then $DS_i[DS_{cl}(A_{FG})] \subseteq DS_{cl}[DS_i(A_{FG})]$

Taking dual soft complement for both side we obtain

$X_{Ds} - [DS_i\ (DS_{cl}(A_{FG}))] \supseteq X_{Ds} - [DS_{cl}\ (DS_i\ (A_{FG})]$

Using The property of the dual soft complement we get

$DS_i[DS_{cl}(X_{Ds} - A_{FG})] \subseteq DS_{cl}[DS_i(X_{Ds} - A_{FG})]$

Hence $X_{Ds} - A_{FG}$ belong to $Su(X_{DS})$

If we assume that B_{F1G1} belong to $Se(X_{DS})$

Then $DS_{cl}[DS_i(B_{F1G1})] \subseteq DS_i(DS_{cl}(B_{F1G1})$

Taking dual soft complement for both side we obtain

$X_{Ds} - [DS_i(DS_{cl}(B_{F1G1}))] \subseteq X_{Ds} - [DS_{cl}(DS_i(B_{F1G1}))]$

Using the property of the dual soft complement we get

$DS_{cl}[DS_i(X_{Ds} - B_{F1G1})] \subseteq DS_i[DS_{cl}\ (X_{Ds} - B_{F1G1})]$

Hence $X_{Ds} - B_{F1G1}$ belong to $Se(X_{DS})$.

Next, we prove about $Co(X_{DS})$ in this case we must say that $DS_i[DS_{cl}(C_{F2G2})]$ and $DS_{cl}[DS_i(C_{F2G2})]$ are non – comparable.

Then $X_{Ds} - [DS_i(DS_{cl}(C_{F2G2}))]$ and $X_{Ds} - [DS_{cl}(DS_i(C_{F2G2}))]$ are also non comparable.

We use reduction absurdum for proving this proposition.

Let $DS_i[DS_{cl}(X_{Ds} - C_{F2G2})]$ and $DS_{cl}[DS_i\ (X_{Ds} - C_{F2G2})]$ be comparable.

Then $X_{Ds} - [X_{Ds} - (C_{F2G2})] = C_{F2G2}$ must belong to $Su(X_{DS})$ or $Se(X_{DS})$

By proved part of this theorem this is contradiction.

2.26 Theorem 2.26

The dual soft set A_{FG} belong to the $Su(X_{DS})$ if

$DS_i\ [DS_{cl}\ (A_{FG}\ )] = DS_i[(DS_{cl}\ DS_i\ (A_{FG}))]$

$DS_{cl}\ [DS_i\ (A_{FG}\ )] = DS_{cl}\ [DS_i(DS_{cl}\ (A_{FG}))]$

On other hand if one of these equalities holds then A_{FG} belong to $Su(X_{DS})$

2.26.1 Proof

Suppose that A_{FG} belong to the $Su(X_{DS})$ Then the following relation holds

$DS_i\ [DS_{cl}\ (A_{FG})] \subseteq DS_{cl}\ [DS_i(A_{FG})]$

taking dual soft interior for both side we get $DS_i\ [DS_{cl}\ (A_{FG})] \subseteq DS_i\ [DS_{cl}\ (DS_i\ (A_{FG}))]$ considering the self- evident relation$DS_i\ [DS_{cl}\ (A_{FG}] \supseteq DS_i\ [DS_{cl}\ (DS_i\ (A_{FG}))]$

we get $DS_i\ [DS_{cl}(A_{FG})] = DS_i[DS_{cl}\ (DS_i\ (A_{FG}))]$

Conversely assume $DS_i\ [DS_{cl}\ (A_{FG})] = DS_i\ [DS_{cl}\ (DS_i\ (A_{FG}))]$

Clearly $DS_{cl}\ [DS_i\ (A_{FG})] \supseteq DS_i\ [DS_{cl}\ (DS_i\ (A_{FG}\ ))] = DS_i\ [DS_{cl}\ (A_{FG})]$

This implies that A_{FG} belong to $Su(X_{DS})$.

Finally, we can conclude that

2.27 Theorem 2.27

Any dual soft set A_{FG}, B_{F1G1} of $Su(X_{DS})$ satisfy the following relation

$DS_i\ [DS_{cl}(A_{FG})] \cap_{DS} DS_i\ [DS_{cl}\ (B_{F1G1})] = DS_i\ [DS_{cl}\ (A_{FG} \cap_{DS} B_{F1G1})]$

$DS_i\ [DS_{cl}\ (A_{FG}) \cup_{DS} DS_i\ [DS_{cl}\ (B_{F1G1})] = DS_i\ [DS_{cl}\ (A_{FG} \cup_{DS} B_{F1G1})]$

3 -The dual soft intersection and dual soft union of any two dual set of $Su(X_{DS})$ belong to $Su(X_{DS})$

2.27.1 Proof

$DS_i[DS_{cl}(A_{FG} \cap_{DS} B_{F1G1})] \subseteq DS_i[DS_{cl}\ (A_{FG})] \cap_{DS} DS_{cl}(B_{F1G1})]$

$= DS_i[DS_{cl}\ (A_{FG}\ )] \cap_{DS} DS_i[DS_{cl}(B_{F1G1})]$

On other hand since A_{FG} , $B_{F1G1} \in Su(X_{DS})$

$DS_i\ (DS_{cl}\ (A_{FG})) \cap_{DS} DS_i(DS_{cl}\ (B_{F1G1})) = DS_i(DS_{cl}\ (DS_i\ (A_{FG}))) \cap_{DS} (DS_{cl}\ (DS_i\ (B_{F1G1})))\ Su(X_{DS}) = DS_i\ (DS_{cl}\ (DS_i\ (A_{FG} \cap_{DS} B_{F1G1})))$ Proposition 2.12 part (5) $\subseteq DS_i\ (DS_{cl}\ (A_{FG} \cap_{DS} B_{F1G1}))$

So $DS_i\ (DS_{cl}\ (A_{FG})) \cap_{DS} DS_i\ (DS_{cl}\ (B_{F1G1})) = DS_i(DS_{cl}\ (A_{FG} \cap_{DS} B_{F1G1}))$.

3 Conclusion

It is possible to study the concept of ideal within these sets through which local function is determined and the effects of dual soft sets on this function are highlighted its most important characteristics.

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