A description of linear mappings in semiprime rings with involution

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Abstract. The main purpose of this paper is to descriptive the action of the linear mappings in semi-prime rings and prime ring with involution. More precisely, we establish some results for centralizer mappings (resp. commuting mappings) of a semi-prime *- ring and *-prime ring $\mathcal{R}$.

1 Introduction

Employing an additive mappings over a rings assist us to investment better. In addition to that, an additive mappings which acts as a derivation over a rings. In fact, a derivation tell us about the structure concerning a rings. Like a ring has a commutative property if and only if the only the inner derivation which works on a ring equal to zero. Moreover, an additive mappings play a role for connecting between a ring and set of matrices with entries in the ring. In fact, there are several authors studied the action of linear mappings on associative rings.

Moreover, the additive mapping L from a ring $\mathcal{R}$ into itself it claims to be a right . Jordan (respect a left Jordan centralizer mapping when $L(p^2) = pL(p)$ (resp. $L(p^2) = L(p)p$). holds for every p member of $\mathcal{R}$. B. Zalar search in [8] that there any Jordan mapping which acts as a ring centralizer (respect left centralizer) over a semi-prime ring which has a 2-torsion free property form a right centralizer (respect left centralizer) mapping . The reference [1] refers to E. Alba's pushed the concept of α-centralizer over $\mathcal{R}$, i.e. the additive mapping L from $\mathcal{R}$ into $\mathcal{R}$ is named a right α-centralizer (respect left $\alpha-$centralizer concerning $\mathcal{R}$ whenever $L(p\omega) = \alpha(p)L(\omega)$ (resp. $L(p\omega) = L(p)\alpha(\omega)$), $\forall\, p,\omega \in \mathcal{R}$, and α acts as an endomorphism over $\mathcal{R}$. In case, L is right and left α-centralizer yield L is normal α-centralizer. Obviously, centralizer mapping is an instance of α−centralizer mapping with the condition that $\alpha = \mathrm{id}_{\mathcal{R}}$. Furthermore, the mapping L from $\mathcal{R}$ to $\mathcal{R}$ which is additive related to a homomorphism α from $\mathcal{R}$ into itself when, $L_a(p) = a\,\alpha(p)$ and $\mathcal{R}_a(p) = \alpha(p)a$ in the case of a fixed element $a \in \mathcal{R}$. $\forall\, p \in \mathcal{R}$, of course L_a from a left α-centralizer mapping and $\mathcal{R}_a$ from a right α-centralizer mapping. Alba's provide that B. Zalar's the same is true for the outcomes α-centralizer. Significant research has been conducted on this subject during the previous few decades (see [1-8], see[1,8-11]. In which can I get more references).

In this article, we descriptive the action of the linear mappings in semi-prime rings and prime ring with involution.

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2 Preliminaries

As a usually, during our work of this project $\mathcal{R}$ will denote an associative ring with a center $Z(\mathcal{R})$. We remember a ring $\mathcal{R}$ with the relation $p\mathcal{R}p = 0$, implies either $p = 0$ or $\omega = 0$. $\forall\, p, \omega \in \mathcal{R}$ a prime ring while when a ring $\mathcal{R}$ satisfies the condition $p\mathcal{R}\, p = 0$, leads to $p = 0$. $\forall\, p \in \mathcal{R}$ is named semi-prime ring . The relation between a prime and semi-prime rings write, as following : Every prime ring implies to semi-prime ring, while the converse is not active always. When we have an integer $n \geq 2$. An associative ring $\mathcal{R}$ has the property that named a n-torsion free whenever $np = 0$, produces that $p = 0$, with note that p belong to $\mathcal{R}$. For any arbitrary elements p and $\omega\, in\, \mathcal{R}$. We use the character $[p, \omega] = p\omega - \omega p$ (resp. $p \circ \omega = p\omega + \omega p$) for the commutaters (resp. anti-commutator). Also, an additive mapping $p \to p^*$ which satisfies the relations (i) $(p\, \omega)^* = \omega^* p^*$ (ii) $(p^*)^* = p$ (iii) $(p + \omega)^* = p^* + \omega^*$. $\forall\, p, \omega$ are a member of $\mathcal{R}$ is named an involution and a ring $\mathcal{R}$ is named a ring with involution (resp. *-ring). Hence , a ring $\mathcal{R}$ with involution is named a *-prime ring when $p\mathcal{R}p = p\,\mathcal{R}\,\omega^* = (0)$ or $p\,\mathcal{R}\,\omega = p^*\,\mathcal{R}\,\omega = (0)$, both case provide two options which are either $p = 0$ or $\omega = 0$. May be somebody asks about the relation between prime ring and *-prime ring. Here, we write that relation as every prime ring with involution is prime ring , but the convers need not be true in general. Moreover, an element a $\in \mathcal{R}$ which satisfies the condition $ap^* = p^* a$ is named normal element of $\mathcal{R}$. When the element $a \in \mathcal{R}$. Achieved the identity $a = p^*$ is named Hermition (or symmetric).

2.1 Lemma (2.1) : [6]

Assume $(\mathcal{R}, +, \cdot)$ is a 2-torsion free semi-prime ring. Suppose that the identity $apb + bpc = 0$ holds. $\forall\, p\, in\, \mathcal{R}$, some of the $a, b, c \in \mathcal{R}$. Then in this instance $(a + c)pb = 0$, satisfied $\forall\, p$ in $\mathcal{R}$.

2.2 Lemma (2.2) : [2]

Suppose $(\mathcal{R}, +, \cdot)$ is a 2-torsion free semi-prime *-ring. Take L as an automorphism over $\mathcal{R}$. When $L: \mathcal{R} \to \mathcal{R}$ acts as an additive mapping satisfying the identity $2L(p^2) = L(p)\,(p^*) + (p^*)\,L(p), \forall\, p$ in $\mathcal{R}$. Then L is a Jordan *-centralizer.

2.3 Lemma (2.3): [12, Theorem3]

Assume p and q are a positive integer. Let $\mathcal{R}$ is a prime ring with char($\mathcal{R}$) = 0 or $p + q + 1$ least than or equal to char($\mathcal{R}$). Assume $L: \mathcal{R} \to \mathcal{R}$ is an additive mapping such that L achieves the identity satisfying the relation $L\left(\mathrm{p}^{p+q+1)}\right) = \mathrm{p}^{p}\, L\,(p)\, \mathrm{p}^{q}\ \forall\, p$ in $\mathcal{R}$. Then L is a two-sided centralizer.

3 The main results

3.1 Theorem (3.1):

Assume $(\mathcal{R}, +, \cdot)$ is a prime ring with involution, and $L : \mathcal{R} \to \mathcal{R}$ acts as a non-zero additive mapping achieves satisfying $L(p\omega) = L(p)\, \omega^*$. $\forall\, \omega, \mathrm{p}$ in $\mathcal{R}$. Then $\mathcal{R}$ has a normal element.

3.1.1 Proof:

From the hypothesis, we own the expression $L(p\omega) = L(p)\ \omega^*\ \forall\ \omega, p$ in $\mathcal{R}$.
Replacing ω by ωz , $z \in \mathcal{R}$.
Then $L(p\omega z) = L(p)\ (\omega z)^*$.
Applying the main condition $L(p\omega) = L(p)\ \omega^*$. We arrive to
$L(p)\ \omega^* z^* = L(p) z^*\ \omega^*$. Moreover,
$L(p)\ \omega^*\ z^* - L(p)\ z^*\ \omega^* = L(p)\ (\omega^*\ z^* - z^*\omega^*) = 0$.
Writing this result as follows:
$L(p)\ [\omega^*,\ z^*] = 0$.
Replacing p by pt . Then
$L(pt)\ [\omega^*,\ z^*] = 0$. Based on definition of L which is $L(pt) = L(p)\ t^*$.
This relation becomes.
$L(p)t^*\ [\omega^*,\ z^*] = 0$.
Replacing t by t^*, we obtain that
$L(p)\ t[\omega^*,\ z^*] = 0$, writing this relation as , $t \in \mathcal{R}$
$L(p)\ \mathcal{R}[\omega^*,\ z^*] = 0$.
Due to that $\mathcal{R}$ is *- prime ring then
Either $L(p) = 0$ or $[\omega^*,\ z^*] = 0$. $\forall\ \omega\ , p$ in $\mathcal{R}$.
Since $L(p)$ is non-zero mapping.
So we have $[\omega^*,\ z^*] = 0$.
Replacing $z = \omega^*$, we find that
$[\omega^*, (\omega^*)^*] = 0$. Clearly, we conclude that
$[\omega^*,\ \omega] = 0$, $\forall\ \omega$ in $\mathcal{R}$. Consequently,
Then $\mathcal{R}$ has a normal elements.

3.2 Corollary (3.2):

$L: \mathcal{R} \to \mathcal{R}$ is a non-zero additive mapping satisfying, $L(p\omega) = L(p)\ \omega^*$ $\forall\ p, \omega$ in $\mathcal{R}$. Assume $\mathcal{R}$ is a *-prime ring with involution. Then p and ω are hermitian elements.

By a similar manner that applied in the proof of Theorem 3.1, one can proof the following results.

3.3 Proposition (3.3):

Suppose $(\mathcal{R}, +, \cdot)$ forms a semi-prime *-ring , $L: \mathcal{R} \to \mathcal{R}$ acts as a non-zero additive mapping satisfying $L(p\omega) = L(p)\ \omega^*$. $\forall\ p, \omega$ in $\mathcal{R}$. Then L acts as a commuting mapping over $\mathcal{R}$.

3.4 Proposition (3.4):

Take $(\mathcal{R}, +, \cdot)$ as a prime ring with involution, and L acts as a non-zero additive mapping satisfying the identity $L(p\omega) = L(p)\ \omega^*$. $\forall\ \omega, p \in \mathcal{R}$.

Then either $\mathcal{R}$ becomes commutative ring or L acts as a central mapping on $\mathcal{R}$.

3.5 Proposition(3.5):

Assume $(\mathcal{R}, +, \cdot)$ is a*-prime ring has non-zero commutates and L acts as an additive mapping $\mathcal{R}$ to $\mathcal{R}$ achieving satisfying. The relation $L(p\omega) = L(p)\ \omega^*$ where $p\ and\ \omega$ belong to $\mathcal{R}$. Then $L(\mathcal{R}) = 0$.

3.6 Theorem (3.6):

Let $\mathcal{p}$ and q as the positive integers and $(\mathcal{R}, +, \cdot)$ be a prime ring with involution. When $\mathcal{R}$ has $\text{char}(\mathcal{R}) = 0$ or $\mathcal{p} + q + 1 \leq \text{char}(\mathcal{R})$, and L is an additive mapping from $\mathcal{R}$ into itself achieving satisfying the relation $L(\text{p}^{\mathcal{p}+q+1}) = (\text{p}^*)^q \text{ L (p) } (\text{p}^*)^{\mathcal{p}}$, $\forall$ p in $\mathcal{R}$. Then L is commuting mapping of $\mathcal{R}$.

3.6.1 Proof:

Utilize the involution on both said of the previous,

we arrive two,

$L(\text{p}^{\mathcal{p}+q+1})^* = ((p^*)^* L(p) (p^*)^*) *$.

Applying the involution mapping and we focus on the right side.

$L(\text{p}^{\mathcal{p}+q+1})^* = (L(p) (p^*)^*) * ((p^*)^q) *$ .Again we applying involution on the term we deduce that.

$(L(p) (p^*)^{\mathcal{p}}) *, ((p^*)^{\mathcal{p}}) * L(\text{p})^* ((p^*)^{\mathcal{p}}) * = \text{p}^* L(p)^* p^q$.

Since $(p^*)^* = \mathcal{R}$. $\forall p$ in $\mathcal{R}$.

$$L(p^{\mathcal{p}+q+1})^* = p^* L(p^*) p^q . \forall p \text{ in } \mathcal{R}. \ \mathcal{p}, q \text{ in } N \quad \text{...(1)}$$

Introduce the new map L from $\mathcal{R}$ into itself where. $L(p) = L(p^*)$. $\forall p$ in $\mathcal{R}$.

Hence L is additive map.

We write the value of $L(p^*)$ in identity (1), we arrive to

$L(\text{p}^{\mathcal{p}+q+1})^* = p^* \text{L (p) } p^q$. $\forall p$ in $\mathcal{R}$.

Based on Lemma 2.3, we note L acts as a two sided-centralizer , i.e.

$L(\text{p}\omega) = \text{p L}(\omega)$ and $L = \text{L(p)}\omega$. $\forall \omega, p$ in $\mathcal{R}$.Obviously, we observe

$L(p)\omega - pL(\omega) = 0$. Replacing ω by p, we note

$[L(p), p] = 0$. $\forall p$ in $\mathcal{R}$.

This expression indicate to L acts as commuting linear mapping on $\mathcal{R}$.

3.7 Theorem (3.7):

Suppose $(\mathcal{R}, +, \cdot)$ is a semi-prime *-ring , $L: \mathcal{R} \to \mathcal{R}$ such that L acts as a centralizer mapping on $\mathcal{R}$. $L(\text{p}^{\mathcal{p}+q+1}) = (p^*)^q L(p) (p^*)^{\mathcal{p}}$. $\forall p$ in $\mathcal{R}$.

Then L is $(\mathcal{p} + q)$-*-commuting of $\mathcal{R}$.

3.7.1 Proof :

From the main relation ,

$L(\text{p}^{\mathcal{p}+q+1}) = p^{\mathcal{p}} L(\text{p})^* p^q$. Since L is centralizer on $\mathcal{R}$.

Then $L(\text{p}^{\mathcal{p}+q+1}) = L(p^{\mathcal{p}}.p^*). \omega^q$. Again we use that L is centralizer ,we note.

$L(\text{p}^{\mathcal{p}+q+1}) = \text{p}^* L(\text{p}^{\mathcal{p}}) \omega^q$, where L is centralize mapping of $\mathcal{R}$.

we observe $L(\text{p}^{q+\mathcal{p}+1}) = p^* L(\text{p}^{\mathcal{p}+q})$.

Using the same previous technique , we find that

$L(\text{p}^{\mathcal{p}+q+1}) = L(\text{p}^{\mathcal{p}\neq q}) p^*$.

Suppose $\mathcal{p} + q = \text{t}$, $t \in \text{Z}$. Thus ,

we derive the new fashion for the previous expression .

$L(\text{p}^{\text{t}+1}) = L(\text{p}^{\text{t}}) \text{p}^*$ and $L(\text{p}^{\text{t}+1}) = \text{p}^* L(\text{p}^{\text{t}})$.

Clearly , we deduce the relation .

$L(\text{p}^{\text{t}}) \text{p}^* = \text{p}^* L(\text{p}^{\text{t}})$.

Moreover, $L(\text{p}^{\text{t}})\text{p}^* - \text{p}^* L(\text{p}^{\text{t}}) = 0$.

Writing this relation as follows.

$[L(\mathrm{p}^*), \mathrm{p}^*] = 0. \forall p$ in $\mathcal{R}$. By other way.
we deliver L is $(\wp + q)$-*-commuting on $\mathcal{R}$, where $t = \wp + q$.

3.8 Theorem (3.8):

Take $L: \mathcal{R} \to \mathcal{R}$ as an additive mapping achieving satisfying $2L(p\omega p) = L(p)\,\sigma(\omega^*)\,\sigma(p^*) + \sigma(p^*)\,\sigma(\omega^*)\,L(p)$. $\forall\, \omega, p \in \mathcal{R}$ so that L utilizer as a homomorphism. Suppose $\mathcal{R}$ a semi-prime $*$-ring has a 2-torsion free property, and σ is an automorphism of $\mathcal{R}$. Then $L(p^2)\,\sigma(p^*) = \sigma(p^*)\,L(\mathrm{p}^2)$. $\forall\, p \in \mathcal{R}$.

3.8.1 Proof :

We discover, according to the hypothesis, that

$$2L(p\omega p) = L(p)\,\sigma(\omega^*)\,\sigma(p^*) + \sigma(p^*)\sigma(\omega^*)\,\mathrm{L(p)}. \quad \forall\, \omega, p \in \mathcal{R}. \quad \text{...(2)}$$

Replacing p by $p + z$ in (2). Then

$$2L(p + \mathrm{z})\omega\,(p + \mathrm{z}) = L(p + \mathrm{z})\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*) + \sigma(\mathrm{p} + \mathrm{z})^*\,\sigma(\omega^*)\,L(p + \mathrm{z}) \quad (3)$$

Simplify the left side of this relation ,we deduce

$2L((p\omega + z\,\omega)(p + z)) = 2L\,(p\omega p + p\omega\, z + z\, q\omega + z\omega z) = 2L\,(p\omega\, z + z\,\omega p + (p\omega p + z\,\omega\, z)) = 2L\,(p\omega\, z + z\,\omega p) + 2L\,(p\omega p + z\,\omega\, z) = 2L\,(p\omega\, z + z\,\omega p) + 2L\,(p\omega p) + 2L\,(z\omega z)$.

According to the main relation, we conclude.

$2L(p\omega z + z\omega p) + L\,(p)\,\sigma(\omega^*)\,\sigma\,(p^*) + \sigma\,(p^*)\,\sigma(\omega^*)\,L\,(p) + L(z)\sigma(\omega^*)\,\sigma(z^*) + \sigma(z^*)\,\sigma\,(\omega^*)L(z) = L\,(p + z)\,\sigma(\omega^*)\,(\sigma\,(p + z)^*) + \sigma((p + z)^*)\,\sigma(\omega)^*\,L(p + z) = L\,(p + z)\,\sigma\,(\omega)\,(\sigma(p + z)^*) + \sigma((p + z)^*)\,\sigma\,(\omega^*)\,L(p + z)$.

Now we focus on the right side.

$(\mathrm{L(p)} + \mathrm{L\,(z)})\,\sigma\,(\omega^*)\,\sigma\,(\mathrm{p}^* + \omega^*)\,\sigma\,(\mathrm{p}^* + \mathrm{z}^*)\,\sigma\,(\omega^*)\,(\mathrm{L\,(p)} + \mathrm{L\,(z)})$. Then

$(\mathrm{L(p)}\,\sigma\,(\omega^*) + \mathrm{L\,(z)}\sigma\,(\omega^*))\,(\sigma\,(\mathrm{p}^* + \mathrm{z}^*)) + \sigma\,(\mathrm{p}^* + \mathrm{z}^*)\,(\sigma\,(\omega^*)\,\mathrm{L(p)} + \sigma(\omega)\,\mathrm{L(z)})$.

Since σ acts as additive mapping i.e. $\sigma\,(\mathrm{p}^*) + \sigma\,(\mathrm{z}^*) = \sigma\,(\mathrm{p}^* + \mathrm{z}^*)$.

Then the right side becomes.

$L(p)\,\sigma\,(\omega^*)\,\sigma(p^*) + L\,(p)\,\sigma(\omega^*)\,\sigma(p) + L(p)\,\sigma(\omega^*)\,\sigma(z^*) + L(z)\,\sigma(\omega^*)\,\sigma(z^*) + \sigma\,(p^*)\,\sigma\,(\omega)\,L\,(p) + \sigma(p^*) + L(z)\,\sigma(\omega^*)\,\sigma(z^*) + \sigma(p^*)\,\sigma(\omega^*)\,L(p) + \sigma(p^*)\,\sigma(\omega^*)\,L(z) + \sigma(z^*)\,\sigma(\omega^*)\,L(p) + \sigma(z^*)\,\sigma(\omega^*)\,L(z)$.

The next step we write this relation by the style.

$2L(p\omega z + z\omega p) = -(L(p)\,\sigma(\omega^*)\,\sigma(z^*) + L(z)\,\sigma(\omega^*)\,\sigma(p^*) + \sigma(p^*)\,\sigma(\omega^*)\,L(z) + \sigma(z^*)\,\sigma(\omega^*)\,L(p))$.

After simply the above equation , we arrive at

$-L(p)\,\sigma(\omega^*)\,\sigma(\mathrm{z}^*) - L(\mathrm{z})\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*) - \sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,L(\mathrm{z}) - \sigma(\mathrm{z}^*)\sigma(\omega^*)\,L(p) + L(p)\,\sigma(\omega^*)\,\sigma(\mathrm{z}^*) + L(\mathrm{z})\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,L(\mathrm{z}) + \sigma(\mathrm{z}^*)\,\sigma(\omega^*)\,L(p) = 0,\ \forall\, p, \omega, \mathrm{z} \in \mathcal{R}$.

Taking p^2 instead of z, we arrive at

$$2L(p\omega p^2 + \mathrm{p}^2\omega p) = L(p)\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*)\,\sigma(\mathrm{p}^*) + L(\mathrm{p}^2)\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,L(\mathrm{p}^2) + \sigma(\mathrm{p}^*)\,\sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,L(p).\ \forall\, p, \omega \in \mathcal{R}\,. \quad (4)$$

Substituting $p\omega + \omega p$ for p in (2), we have

$$2L(p\omega\mathrm{p}^2 + \mathrm{p}^2\omega p) = L\,(p)\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*)\,\sigma(\mathrm{p}^*) + L\,(p)\,\sigma(\mathrm{p}^*)\,\sigma(\omega)\,\sigma\,(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*)\,L(p) + \sigma(\mathrm{p}^*)\,\sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,L(p),\ \forall\, p, \omega \in \mathcal{R}. \quad (5)$$

By comparing (4) and (5), we get

$L(\mathrm{p}^2)\,\sigma(\omega^*)\,\sigma\,(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,L\,(\mathrm{p}^2) = L\,(p)\,\sigma(\mathrm{p})\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*)\,L(p)$, and so

$$(L(\mathrm{p}^2) - L(p)\,\sigma(\mathrm{p}^*))\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\,\sigma(\omega^*)\,(L(\mathrm{p}^2) - \sigma(\mathrm{p}^*)\,L(p)) = 0.\ \forall\, p, \omega \in \mathcal{R}. \quad (6)$$

Putting ω^* for in (5), yields that

$(L(\mathrm{p}^2) - L(p)\,\sigma(\mathrm{p}^*)\,\sigma(\omega)\,\sigma(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\,\sigma(\omega)\,(L(\mathrm{p}^2) - \sigma(\mathrm{p}^*)\,L(p)) = 0.\ \forall\, p, \omega \in \mathcal{R}.$

Since σ is an epimorphism of $\mathcal{R}$, we have

$\big(L(\mathrm{p}^2) - L(p)\sigma(\mathrm{p}^*)\big)\omega\,\sigma(\mathrm{p}^*) + \sigma(\mathrm{p}^*)\omega\,\big(L(\mathrm{p}^*) - \sigma(\mathrm{p}^*)L(p)\big) = 0.$

By Lemma 2.1, we conclude that

$$\big(2L(\mathrm{p}^2) - L(p)\sigma(p^*) - \sigma(p^*)L(p)\big)\omega\,\sigma(p^*) = 0.\ \forall\, p, \omega \in \mathcal{R}. \quad (7)$$

Define $\mathcal{S}(p) = 2L(\mathrm{p}^2) - L(p)\sigma(p^*) - \sigma(p^*)L(p)$.

Hence (7), can be rewritten as.

$$\mathcal{S}(p)\,\omega\,\sigma(\mathrm{p}^*) = 0.\ \forall\, p, \omega \in \mathcal{R}. \quad (8)$$

Substituting $\sigma(\mathrm{p}^*)\,\omega\,\mathcal{S}(p)$ for in (8), we get

$\mathcal{S}(p)\,\sigma(\mathrm{p}^*)\,\omega\,\mathcal{S}(p)\,\sigma(\mathrm{p}^*) = 0.\ \forall\, p, \omega \in \mathcal{R}$.

Using $\mathcal{R}$ is semi-prime ring, deduce that

$$\mathcal{S}(p)\,\sigma(\mathrm{p}^*) = 0, \forall p \in \mathcal{R}. \quad (9)$$

Left multiplying (8) by $\sigma(\mathrm{p}^*)$ and right multiplying (9) by $\mathcal{S}(p)$, we have

$\sigma(\mathrm{p}^*)\,\mathcal{S}(p)\,\mathcal{S}\,\sigma(\mathrm{p}^*)\,\mathcal{S}(p) = 0.\ \forall\, p \in \mathcal{R}.$

Then $\sigma(\mathrm{p}^*)\,\mathcal{S}(p)\,\mathcal{R}\,\sigma(\mathrm{p}^*)\,\mathcal{S}(p) = 0.$

By the semiprimeness of $\mathcal{R}$, arrive at

$$\sigma(\mathrm{p}^*)\mathcal{S}(p) = 0.\ \forall\, p \in \mathcal{R}. \quad (10)$$

Replacing p by $p + \omega$ in (9), and using this we arrive to

$\mathcal{S}(p + \omega)\sigma(\mathrm{p}^* + \omega^*) = 0,$ and so

$(\mathcal{S}(p) + \mathcal{S}(\omega)\,2L(p\omega + q\omega) - L(p)\,\sigma(\omega^*) - L(\omega)\,\sigma(\mathrm{p}^*) - \sigma(\mathrm{p}^*)$

$L(p) - \sigma(\omega^*)\,L(p)\sigma(\mathrm{p}^* + \omega^*) = 0.$ That is

$\mathcal{S}(p)\,\sigma(\mathrm{p}^*) + \mathcal{S}(\omega)\,\sigma(\mathrm{p}^*) + \mathcal{N}(p, \omega)\,\sigma(\mathrm{p}^*) + \mathcal{S}(p)\,\sigma(\omega^*) + \mathcal{S}(\omega)\,\sigma(\omega^*) + \mathcal{N}(p, \omega)\,\sigma(\omega^*) = 0,\ \forall p, \omega \in \mathcal{R}.$

Where $\mathcal{N}(p, \omega) = 2L(p\omega + q\omega) - L(p)\,\sigma(\omega^*) - L(\omega)\,\sigma(\mathrm{p}^*) - \sigma(\mathrm{p}^*)L(\omega) - \sigma(\omega^*)\,L(p).$

Using (10), in the last equation we see that

$$\mathcal{S}(p)\,\sigma(\omega^*) + \mathcal{S}(\omega)\,\sigma(\mathrm{p}^*) + \mathcal{N}(p, \omega)\,\sigma(\mathrm{p}^*) + \mathcal{N}(p, \omega)\,\sigma(\omega^*) = 0.\ \forall\, p, \omega \in \mathcal{R}. \quad (11)$$

Substituting $-p$ for p in (11), and we find that

$(\mathcal{S}(-p)\,\sigma(\omega^*) + \mathcal{S}(\omega)\,\sigma(-\mathrm{p}^*) + \mathcal{N}(-p, \omega)\,\sigma(-\mathrm{p}^*) + \mathcal{N}(-p, \omega)\,\sigma(\omega^*) = (0).$

Applying the relation $\mathcal{S}(-p) = \mathcal{S}(p)$ and $\mathcal{N}(-p, \omega) = -\mathcal{N}(p, \omega).$

We obtain

$$\mathcal{S}(p)\,\sigma(\omega^*) - \mathcal{S}(\omega)\,\sigma(\mathrm{p}^*) + \mathcal{N}(p, \omega)\,\sigma(\mathrm{p}^*) - \mathcal{N}(p, \omega)\,\sigma(\omega^*) = 0.\ \forall\, p, \omega \in \mathcal{R}. \quad (12)$$

Now combining (10) and (12), we get

$$\mathcal{S}(p)\,\sigma(\omega^*) + \mathcal{N}(p, \omega)\,\sigma(\mathrm{p}^*) = 0.\ \forall\, p, \omega \in \mathcal{R}. \quad (13)$$

Multiplying (13) from the right by $\mathcal{S}(p)$ and according to (10), that is the outcome we get.

$\mathcal{S}(p)\sigma(\omega^*)\mathcal{S}(p) = 0.\ \forall\, p, \omega \in \mathcal{R}.$

Writing ω^* as an alternative to ω and σ is an epimorphism of $\mathcal{R}$, we currently have.

$\mathcal{S}(p)\,\omega\,\mathcal{S}(p) = 0.\ \forall\, p, \omega \in \mathcal{R}.$

Consequently $\mathcal{S}(p)\,\mathcal{R}\,\mathcal{S}(p) = 0.$

Due to semiprimeness of $\mathcal{R}$.

Observe that $\mathcal{S}(p) = 0.\ \forall\, p \in \mathcal{R}.$

Obviously, $2L(\mathrm{p}^2) - L(p)\,\sigma(\mathrm{p}^*) - \sigma(\mathrm{p}^*)\,L(p) = 0.\ \forall\, \mathrm{p} \in \mathcal{R}$.

Then $[2L(\mathrm{p}^2), \mathrm{r}] - [L(p)\,\sigma(\mathrm{p}^*), \mathrm{r}] - [\sigma(\mathrm{p}^*)\,L(p), \mathrm{r}] = 0. \forall\, r, p \in \mathcal{R}.$

Where $2L(\mathrm{p}^2) - L(p)\,\sigma(\mathrm{p}^*)\,L(p) \in \mathrm{Z}(\mathcal{R})$.

Replacing r by $L(p)$, we find that.

$2[L(\mathrm{p}^2), L(p)] - [L(p)\,\sigma)\mathrm{p}^*), L(p)] - [\sigma(\mathrm{p}^*)\,L(p), L(p)] = 0.$

Due to L acts as a homomorphism , this relation modifies to

$-L(p)[\sigma(\mathrm{p}^*), L(p)] - [\sigma(\mathrm{p}^*), L(p)]\,L(p) = 0. \forall\, p \in \mathcal{R}$. Moreover,

$= L(p)\,\sigma(p^*)\,L(p) + L(p)^2\sigma(p^*) - \sigma(p^*)\,L(p)^2 + L(p)\,\sigma(p^*)\,L(p) = 0.$ Clearly, we deduce .

$L(\mathrm{p})^2\sigma(\mathrm{p}^*) - \sigma(\mathrm{p}^*)\,L(\mathrm{p})^2 = 0.$

Then $L(\mathrm{p})^2\sigma(p) = \sigma(\mathrm{p}^*)\,L(\mathrm{p})^2$.Since L is a homomorphism.

The last expression becomes.

$L(p)^2\sigma(p^*) = \sigma(p^*)\,L(p^2).\ \forall\, p \in \mathcal{R}.$

This is requirement result .

3.9 Corollary 3.9:

If $L: \mathcal{R} \to \mathcal{R}$ is an additive mapping satisfying $2L(p\omega p) = L(p)\,\sigma(\omega)\,\sigma(p^*) + \sigma(p)\,\sigma(\omega^*)\,L(p).\ \forall\, \omega,\ p \in \mathcal{R}$. Take σ as an automorphism over $\mathcal{R}$. Assume $\mathcal{R}$ is a 2-torsion free *-prime ring. Then either $\sigma(\mathrm{p}^*) = 0$ or L is a Jordan $\sigma - *$centralizer.

3.9.1 Proof:

Applying the same strategy which used in the proof of Theorem 3.8

specific relation (13), we obtain

$\big(2L(\mathrm{p}^2) - L(p)\sigma(p) - \sigma(p^*)L(p)\big)\mathcal{R}\,\sigma(p^*) = (0).$

Due to $\mathcal{R}$ is a prime ring, we arrive to options.

The first option $2L(\mathrm{p}^2) - L(p)\,\sigma(p^*) - \sigma(p^*)L(p) = 0. \forall\, p \in \mathcal{R}$.

In other words , we deduce

L is a Jordon $\sigma - *$centralizer by Lemma 2.1.

While the second option is $\sigma(\mathrm{p}^*) = 0,\ \forall \mathrm{p} \in \mathcal{R}$.

In the next step , if we restrict the action of σ as a non-zero mapping .

Then Corollary 3.9, supply the following result.

3.10 Corollary (3.10):

Suppose $\mathcal{R}$ is *-prime ring has a 2-torsion free. Suppose that σ in automorphnism mapping of $\mathcal{R}$ such that σ is a non –zero mapping. Take $L: \mathcal{R} \to \mathcal{R}$ as an additive mapping achieving satisfying $2L(p\,\omega p) = L(p)\,\sigma(\omega^*)\,\sigma(\mathrm{p}^*) + \sigma(p^*)\,\sigma(\omega^*)\,L(p).\ \forall\, p, \omega \in \mathcal{R}.$

Then L is a Jordan $\sigma - *$centralizer of $\mathcal{R}$

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4 Conclusions

In this work, we proved descriptive the action of the linear mapping : $\mathcal{R} \to \mathcal{R}$ an associative ring ($\mathcal{R}$, + , .), specific semiprime rings and prime ring with involution. We establish some results for centralizer mappings $[L(p), p] \in Z(\mathcal{R})$ (resp. commuting mappings $[L(p), p] = (0),\ \forall\, p \in \mathcal{R}$ of a semi-prime *-ring and *-prime ring $\mathcal{R}$.

When $*$ acts as an additive mapping $p \to p^*$ satisfying $(p\omega)^* = \omega^* p^*$ and $(p^*)^* = p.\ \forall\, p, \omega \in \mathcal{R}$, is named an involution. Additionally, it is named a *-ring. An associative ring with

involution is named a prime ring with involution when $p\omega = p\omega^* = (0)$ or $p\omega = p^*\omega = (0)$. leads that either $p = 0$ or $\omega = 0$. As a matter of fact, every prime ring with an involution is a *-prime ring. Unfortunately

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