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Estimation of the parameters of triple modular redundancy system under step partially accelerated life tests using Frechet distribution.

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Abstract: Triple modular redundancy system (TMR) is repetition of important components or functions of the system. It aims to increase the reliability and reducing the failure rate. This study is focused on the estimating of the distribution's parameters of the "TMR system" under the partly life test with gradual stress, as it included every component in the system that follows The Frechete distribution. This research aimed to analysis the reliability of this triple modular system. The optimal time for stress change was determined using two criteria according to the accelerated stress strategy under the new distribution, as well as estimating the reliability of the system by using the maximum likelihood (MLE) method. The efficiency of the "MLE" in estimate the parameters of the new distribution is one of the most important conclusions in this study. Therefore, the design of the "TMR" system can have a lengthy lifetime and higher safety, reducing the risks of unexpected failure and the economic losses.

1 Introduction

Triple modular redundancy (TMR) is a powerful technique characterized by a stable and reliable service level in areas of life such as the electrical power distribution networks and patient support device. This study aims to analysis the reliability of the "TMR" system according to the accelerated pressure strategy under the new distribution. Due to the increasing the errors occurring in the system, it is necessary to reduce the errors or failures through the TMR system. In (2020), Arifeen and et al, they performed the effect of a (TMR) implementation on the reliability of the system was tested [1]. In (2020) Cannon and et al, performed (TMR) was commonly utilized to raise the reliability and the mean time to the failure of the system [2]. In (2021) Katkoori and et al, performed study partial evaluation depends on the "TMR" for single event upset mitigation [3]. As well as the utilizing of the partially "ALT" were used in product design process by exposing experimental units to pressure conditions more severe than the conditions they faced in the normal use. Accelerated life tests were used in many fields such as the electronics, medicine, and aviation industries. In (2019) Nassr and El Haroun, they created four performed studies on the stress partially "ALT" were studied according to the exponentiated "Weibull distribution" [4]. In (2022) Kamal and et al, they performed the a estimation of the parameter depend on the censored

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data with the partly "ALT" for the hybrid systems because the anonymous failure causes [5]. In (2022) Zhang and et al, they created a study about the "bayesian reliability analysis" for copula based "step stress partially accelerated" dependent competing risk sample [6]. In (2022) Nassar and et al, they performed analysis of "modified kies exponential distribution" with fixed stress partly "ALT" with (Kind-2) censoring [7]. In (2023) Al-Essa and et al, they created eight studies on the reliability analysing of the TMR system underof the step partially "ALT" by using" Lomax distribution" [8].

In this system the "MLE" was applied for estimating the three amounts of the scale parameters in three various cases for the parameters of the" Frechete distribution"and finding the best estimator for the "TMR" system by using the simulation for the purpose of comparing distribution parameters by using the MSE.

2 Design of the triple modular redundancy system

The system works correctly when at least two of the three components must be running in order to continue in the work [9] . The component (U) are repeated three times and these three components work in parallel. The component (V) meets the outcomes of the three components (U) as shown in Figure 1. If there is a malfunction in one of the three, components the system's operation is allowed to continue without interruption to the point that the failure of the units cannot be felt. Therefore, the TMR system is suitable for cases of the temporary failure due to this property.

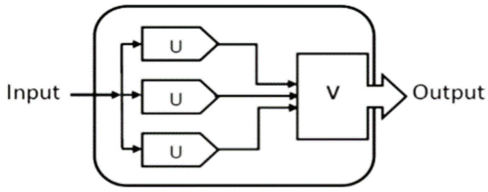

Fig. 1. Structure of system TMR [11].

3 Accelerated life tests

There are three accelerated life tests techniques [10]. The first technicality is called to as continuous pressure "ALT", through which pressure is applied at a constant level throughout the product's operation. The second technique is through gradual pressure [11]. The stress well be product during the operation increases over the time. The third technique is a stage of changing for the gradual voltage of the stress level after appearing a specific number of failures or after a limited period of time.

4 .1. Frechete distribution

The Frechete distribution considered as a one of the most well-known probability distributions [12]. It was named the French mathematician (Maurice Rene Frechete). It is used in data modeling, analysis, applied statistics and other related fields.

The "(PDF)" and "(CDF)" are given by

$$f(t;\theta,\alpha) = \frac{\alpha}{t}\left(\frac{\theta}{t}\right)^{\alpha} e^{-\left(\frac{\theta}{t}\right)^{\alpha}} \quad (1)$$

$$F(t) = e^{-\left(\frac{\theta}{t}\right)^{\alpha}} \quad (2)$$

also the "(RF)" and "(HRF)" are given by

$$R(t) = 1 - e^{-\left(\frac{\theta}{t}\right)^{\alpha}} \quad (3)$$

$$h(t) = \frac{\frac{\alpha}{t}\left(\frac{\theta}{t}\right)^{\alpha} e^{-\left(\frac{\theta}{t}\right)^{\alpha}}}{1-e^{-\left(\frac{\theta}{t}\right)^{\alpha}}} \quad (4)$$

Where θ> 0 is the (shape parameters) and α> 0 is the(scale parameters)of the distribution.

4 .2. K-out of-N systems

The most commonly utilized reliability models in engineering fields are "K-out of-N" systems, which are either binary or sequential systems. It is considered as A more elastic tool for modeling engineering systems, and it is a special case of parallel replication when at least (K – components) succeed from the sum of the other components.

$$R_k - \text{out} - \text{of} - \text{N(t)} = \sum_{k=i}^{n} \binom{n}{i} [R(t)]^i [1 - R(t)]^{n-i} \quad (5)$$

4 .3. Reliability of the "TMR"system

Suppose that three components (C1, C2, and C3) are constructed in a triple modular redundancy system which is set up to permit plurality voting. For the system to the function efficaciously, at least (two of three) components should be in operation. Suppose that "T is a random variable "(r.v) that represented the "TMR" system's lifetime. If the component reliability is (1), the reliability of the "TMR" system at the time (t), as follow:

$$R_{TMR} = R_{C1(t)}R_{C2}\left(1 - R_{C3(t)}\right) + R_{C1(t)}\, R_{C3}\left(1 - R_{C2(t)}\right) + R_{C3(t)}\, R_{C2}\left(1 - R_{C1(t)}\right) + R_{C1(t)}\, R_{C2(t)}\, R_{c3(t)}$$

$R_{Ci(t)} = R(t) \qquad i = 1,2,3$,

By substitution (k=2), (n=3) in equation (5) then the result will be:

$$R_{TMR(t)} = 3R^2(t) - 2R^3(t) \quad (6)$$

By the equation (6) it will possible to get a "PDF" and "CDF" to the "TMR" system.

$$F_{TMR(t)} = 2R^3(t) - 3R^2(t) + 1 \quad (7)$$

$$f_{TMR(t)} = 6f_{(t)}[1 - R(t)]R(t) \quad (8)$$

Using the equation (1), equation (3) and form (θ, α> 0) the "CDF", "PDF", "RF", "HR "of the "TMR" system are given:

$$F_{TMR(t)} = 3\, e^{-2\left(\frac{\theta}{t}\right)^{\alpha}} - 2e^{-3\left(\frac{\theta}{t}\right)^{\alpha}} \quad (9)$$

$$f_{TMR(t)} = 6\,\frac{\alpha}{t}\,\left(\frac{\theta}{t}\right)^{\alpha}\left[e^{-2\left(\frac{\theta}{t}\right)^{\alpha}} - e^{-3\left(\frac{\theta}{t}\right)^{\alpha}}\right] \quad (10)$$

$$R_{TMR(t)} = 1 - 3\, e^{-2\left(\frac{\theta}{t}\right)^{\alpha}} + 2e^{-3\left(\frac{\theta}{t}\right)^{\alpha}} \quad (11)$$

$$h_{TMR(t)} = \frac{6\,\frac{\alpha}{t}\,\left(\frac{\theta}{t}\right)^{\alpha}\left[e^{-2\left(\frac{\theta}{t}\right)^{\alpha}} - e^{-3\left(\frac{\theta}{t}\right)^{\alpha}}\right]}{1-3\, e^{-2\left(\frac{\theta}{t}\right)^{\alpha}} + 2e^{-3\left(\frac{\theta}{t}\right)^{\alpha}}} \quad (12)$$

4 .4. Description of the model of (TMR) system

This part is for applying the step stress partially accelerated life tests to the "TMR" system whose life "T" is considered a random variable with couvert (t) of "CDF" like the equation (9). The "PDF, SF and HRF" of "T" are provided, by equations "10", "11" and "12". Two techniques are utilized to raise the pressure in the step pressure "PALTs", one of them is called "tampered random variable" [13] and the other is called "tampered failure rate TFR" [14]. For the "TMR" system with the "TFR" technique, as soon as an alteration time point "η" is reached, the pressure raises and remains fixed. This would be done by multiply the basically "HRF $h_{TMR(t)}$" by an indefinite factor (β> 1) which may depend on the "η". Let h^*(t) denote the overall "HRF"in "PALT". Then "TFR" model is:

$$h^*(t) = \begin{cases} h_{TMR(t)} & 0 < t \le \eta \\ \beta h_{TMR(t)} & \eta < t < \infty, (\beta > 1) \end{cases} \tag{13}$$

$$R^*_{(t)} = \begin{cases} R_{1(t)} = e^{\left[-\int_0^t h\, TMR(u)\, du\right]} & 0 < t \le \eta \\ R_{2(t)} = e^{\left[-\int_0^\eta h\, TMR(u) du - \int_\eta^t \beta\, h\, TMR(u) du\right]} & \eta < t < 0 \end{cases} \tag{14}$$

The function $R_2(t)$ defined in (14) is:

$$R_{2(t)} = e^{\left[-\int_0^\eta \frac{f\,TMR(u)}{R\,TMR(u)} du - \int_\eta^t \beta \frac{f\,TMR(u)}{S\,TMR(u)} du\right]} = \left[R_{TMR(t)}\right]^\beta \left[R_{TMR(\eta)}\right]^{1-\beta} \tag{15}$$

By replacing (10) in (15):

$$\mathrm{R}_{2(t)} = \left[1 - 3\,\mathrm{e}^{-2\left(\frac{\theta}{t}\right)^\alpha} + 2\mathrm{e}^{-3\left(\frac{\theta}{t}\right)^\alpha}\right]^\beta * \left[1 - 3\,e^{-2\left(\frac{\theta}{\eta}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{\eta}\right)^\alpha}\right]^{1-\beta} \tag{16}$$

Based on the (10), (14),(16), of T under(PALT):

$$R^*\,(t) = \begin{cases} R_{1(t)} = \left[1 - 3\,e^{-2\left(\frac{\theta}{t}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right] & 0 < t \le \eta \\ R_{2(t)} = \left[1 - 3\,e^{-2\left(\frac{\theta}{t}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right]^\beta * \left[1 - 3\,e^{-2\left(\frac{\theta}{\eta}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{\eta}\right)^\alpha}\right]^{1-\beta} & \eta < t < \infty \end{cases} \tag{17}$$

The (CDF), (PDF), (HRF) are given:

$$\mathrm{F}^*(\mathrm{t}) \begin{cases} F_{1(t)} = 3\,e^{-2\left(\frac{\theta}{t}\right)^\alpha} - 2e^{-3\left(\frac{\theta}{t}\right)^\alpha} & 0 < t \le \eta \\ F_{2(t)} = 1 - \left[1 - 3\,e^{-2\left(\frac{\theta}{t}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right]^\beta * \left[1 - 3\,e^{-2\left(\frac{\theta}{\eta}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{\eta}\right)^\alpha}\right]^{1-\beta} & \eta < t < \infty \end{cases} \tag{18}$$

$$\mathrm{f}^*(\mathrm{t}) = \begin{cases} f_{1(t)} = 6\,\frac{\alpha}{t}\,\left(\frac{\theta}{t}\right)^\alpha \left[e^{-2\left(\frac{\theta}{t}\right)^\alpha} - e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right] & 0 < t \le \eta \\ f_{2(t)} = 6\frac{\alpha}{t}\,\beta\,\left(\frac{\theta}{t}\right)^\alpha \left[e^{-2\left(\frac{\theta}{t}\right)^\alpha} - e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right]\left[1 - 3\,e^{-2\left(\frac{\theta}{t}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right]^{\beta-1} \left[1 - 3\,e^{-2\left(\frac{\theta}{\eta}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{\eta}\right)^\alpha}\right]^{1-\beta} & \eta < t < \infty \end{cases} \tag{19}$$

$$h^*(t) = \begin{cases} h_{1(t)} = \dfrac{6\,\frac{\alpha}{t}\,\left(\frac{\theta}{t}\right)^\alpha\left[e^{-2\left(\frac{\theta}{t}\right)^\alpha} - e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right]}{1 - 3\,e^{-2\left(\frac{\theta}{t}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{t}\right)^\alpha}} & \mathbf{0 < t \le \eta} \\ h_{2(t)} = \dfrac{6\,\frac{\alpha}{t}\beta\,\left(\frac{\theta}{t}\right)^\alpha\left[e^{-2\left(\frac{\theta}{t}\right)^\alpha} - e^{-3\left(\frac{\theta}{t}\right)^\alpha}\right]}{1 - 3\,e^{-2\left(\frac{\theta}{t}\right)^\alpha} + 2e^{-3\left(\frac{\theta}{t}\right)^\alpha}} & \boldsymbol{\eta < t < \infty} \end{cases} \tag{20}$$

4 .5. Mean residual lifetime (MRL)

For the aims of studying the survival analysis, "MRL" is decisive. By supposition the system has lived up to till time "$t_\circ$". "MRL" is defined as the expected residual lifetime.

$$m_{(t_\circ)} = E\,[(T - t_\circ)/T > t_\circ] = \frac{1}{R^*(t_\circ)} \int_{t_\circ}^\infty R^*(t)\,dt$$

$$\text{If, } t_\circ \le \eta, m_{(t_\circ)} = \frac{1}{R_{1(t_\circ)}}\left[\int_{t_\circ}^\eta R_1(t)dt + \int_\eta^\infty R_2(t)dt\right]$$

$$=\frac{1}{\left[1-3\,e^{-2\left(\frac{\theta}{t_\circ}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t_\circ}\right)^{\alpha}}\right]}\left[\int_{t_\circ}^{\eta}\left(1-3\,e^{-2\left(\frac{\theta}{t}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t}\right)^{\alpha}}\right)dt+\int_{\eta}^{\infty}\left[1-3\,e^{-2\left(\frac{\theta}{t}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t}\right)^{\alpha}}\right]^{\beta}\left[1-3\,e^{-2\left(\frac{\theta}{\eta}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{\eta}\right)^{\alpha}}\right]^{1-\beta}dt\right]$$

$$\text{If, } t_\circ>\eta,\ m_{(t_\circ)}=\frac{1}{R_{2(t_\circ)}}\int_{t_\circ}^{\infty}R_2(t)\,dt$$

$$=\frac{1}{\left[1-3\,e^{-2\left(\frac{\theta}{t_\circ}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t_\circ}\right)^{\alpha}}\right]^{\beta}}\int_{t_\circ}^{\infty}\left[1-3\,e^{-2\left(\frac{\theta}{t}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t}\right)^{\alpha}}\right]^{\beta}dt \qquad (21)$$

5 Maximum likelihood estimation

Suppose that t=($t_1 < t_2 < t_3 < \cdots < t_{n1} \le \eta < t_{n1+1} < \cdots < t_n$) is an arranged observed random sample is taken from a population with eq(19), where the n_1 (n_2 =n $-n_1$) is a random number referring the number of fail with normal accelerated stress condition .

$$L(t;\theta,\alpha,\beta)=\left[\prod_{i=1}^{n}f_1(t_i)\right]*\left[\prod_{i=n+1}^{n1}f_2(t_i)\right] \qquad (22)$$

By substitution the eq (19) into eq (22) with taking the logarithm and form the outcome it is possible to obtain the (MLE) parameters.

$$\text{nL}=\text{nLn}(6\alpha)+n\alpha Ln(\theta)-(1+\alpha)\sum_{i=1}^{n}Ln(t_i)+\sum_{i=1}^{n}Ln\left(e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}-e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\right)+n_1 Ln(\beta)+(\beta-1)\sum_{i=n+1}^{n_1}\text{Ln}\left(1-3\,e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\right)^{\beta-1}+n_1(1-\beta)\text{Ln}\left(1-3\,e^{-2\left(\frac{\theta}{\eta}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{\eta}\right)^{\alpha}}\right) \qquad (23)$$

By taking the first derivative to eq (23) to $(\beta,\alpha,\theta)$and equal it to the zero:

$$\frac{\partial lnl}{\partial\alpha}=\frac{n}{\alpha}+nLn\theta-\sum_{i=1}^{n}Ln(t_i)+\sum_{i=1}^{n}\frac{\left[6e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha}Ln\left(\frac{\theta}{t_i}\right)-6e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha}Ln\left(\frac{\theta}{t_i}\right)\right]}{\left(e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}-e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\right)}$$
$$\sum_{i=n+1}^{n_1}\frac{(\beta-1)\left[6e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha}Ln\left(\frac{\theta}{t_i}\right)-6e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha}Ln\left(\frac{\theta}{t_i}\right)\right]}{\left(1-3\,e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\right)}+$$
$$\sum_{i=n+1}^{n_1}\frac{(1-\beta)\left[6e^{-3\left(\frac{\theta}{\eta}\right)^{\alpha}}\left(\frac{\theta}{\eta}\right)^{\alpha}Ln\left(\frac{\theta}{\eta}\right)-6e^{-2\left(\frac{\theta}{\eta}\right)^{\alpha}}\left(\frac{\theta}{\eta}\right)^{\alpha}Ln\left(\frac{\theta}{\eta}\right)\right]}{\left(1-3\,e^{-2\left(\frac{\theta}{\eta}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{\eta}\right)^{\alpha}}\right)}=0 \qquad (24)$$

$$\frac{\partial\text{lnl}}{\partial\theta}=\frac{\text{n}\alpha}{\theta}+\sum_{i=1}^{n}\frac{\left[6e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha-1`}\left(\frac{\alpha}{t_i}\right)-6e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha-1}\left(\frac{\alpha}{t_i}\right)\right]}{\left(e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}-e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\right)}$$

$$+\sum_{i=n+1}^{n_1}\frac{(\beta-1)\left[6e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha-1}\left(\frac{\alpha}{t_i}\right)-6e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}\left(\frac{\theta}{t_i}\right)^{\alpha-1}\left(\frac{\alpha}{t_i}\right)\right]}{\left(\mathbf{1-3\,e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}}\right)}+$$

$$\sum_{i=n+1}^{n_1}\frac{(1-\beta)\left[6e^{-3\left(\frac{\theta}{\eta}\right)^{\alpha}}\left(\frac{\theta}{\eta}\right)^{\alpha-1}\left(\frac{\alpha}{\eta}\right)-6e^{-2\left(\frac{\theta}{\eta}\right)^{\alpha}}\left(\frac{\theta}{\eta}\right)^{\alpha-1}\left(\frac{\alpha}{\eta}\right)\right]}{\left(\mathbf{1-3\,e^{-2\left(\frac{\theta}{\eta}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{\eta}\right)^{\alpha}}}\right)}=0 \qquad (25)$$

$$\frac{\partial Lnl}{\partial\beta}=\frac{n1}{\beta}+\sum_{i=n+1}^{n1}Ln\left(\mathbf{1-3\,e^{-2\left(\frac{\theta}{t_i}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{t_i}\right)^{\alpha}}}\right)-n1Ln\left(\mathbf{1-3\,e^{-2\left(\frac{\theta}{\eta}\right)^{\alpha}}+2e^{-3\left(\frac{\theta}{\eta}\right)^{\alpha}}}\right)=0 \qquad (26)$$

The equations (24), (25)and(26) cannot be solved by using ordinary methods because they are non-linear equations .Therefore ,they are solved by using the numerical method to obtain the coefficients MEL

6 Simulation experiment

A simulation-study was implemented to evaluate the MLE of the parameter of the new TMR system. A set of values were assumed for each parameter of the distribution, as in Table (1), and various sample sizes (25, 50, 100, 200, 400) are used. MSE for each parameter estimation was calculated as in the table (1)

Table 1. Estimated values and MSE of the parameters (α, β, θ) by using Mle method.

N	PARAMETAR			ESTMATOR			MSE		
	α	θ	β	α	θ	β	α	θ	β
25	0.9	0.5	0.5	0.9387241	0.5620369	0.625473	0.13434841	0.03048197	0.07027071
50				0.8723093	0.5308339	0.5510608	0.02637944	0.00990898	0.01301564
100				0.8796422	0.5219953	0.5276763	0.00537748	0.00259632	0.00285985
200				0.8808472	0.5145821	0.5167204	0.00124128	0.00065765	0.00068422
400				0.8862817	0.5099508	0.5103948	0.00030273	0.00016348	0.0001669
25	0.5	0.9	0.5	0.4190301	1.4859924	0.7132533	0.01420335	0.45804767	0.05662677
50				0.4463092	1.2007315	0.607223	0.00320866	0.10449746	0.01225453
100				0.4720108	1.041941	0.5511533	0.00082607	0.0219917	0.0027208
200				0.4857551	0.9688611	0.5248899	0.00020442	0.00493556	0.00062677
400				0.4928636	0.9340261	0.5122779	0.00005112	0.00118028	0.00015158
25	0.5	0.5	0.5	0.4339539	0.7876431	0.686007	0.01784283	0.1264834	0.0521083
50				0.4533782	0.6433901	0.5920551	0.00404288	0.02833993	0.01110652
100				0.4704906	0.5771799	0.5491894	0.00091221	0.00657309	0.00251965
200				0.4850333	0.5374057	0.5239221	0.00022632	0.001465	0.00057993
400				0.4925241	0.5183022	0.5117416	0.00005634	0.0003446	0.00013916
25	2	2.5	1.5	1.683076	3.386863	2.123481	0.16719065	0.86135983	0.46396901
50				1.807072	2.965253	1.810462	0.05384675	0.22473938	0.10475149
100				1.886227	2.733275	1.656524	0.01513809	0.05561913	0.02553673
200				1.939089	2.616851	1.579692	0.00400798	0.0138356	0.00649443
400				1.96855	2.558451	1.54018	0.00103349	0.00344515	0.00163392
25	2.5	1.5	3.5	1.599139	1.647319	3.846727	0.87660847	0.02707685	0.74071241
50				1.993837	1.56225	3.424964	0.28050215	0.004429	0.05386627
100				2.248134	1.528078	3.454054	0.06766187	0.00083494	0.00882739
200				2.374848	1.51368	3.45671	0.01624459	0.00019304	0.00242408
400				2.438847	1.506623	3.47488	0.00381436	0.00004448	0.00066508

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Table 1 shows the efficiency of the maximum likelihood method for all estimators. According to the first and fifth experiments, the parameter α is characterized by having the lower (MSE) compared to the other parameters. However, in the second, third, and fourth experiments, the parameter θ is characterized by having the lower (MSE) compared to the other parameters. In general, it is possible to calculate all of the estimators that have a relatively small mean square to the extent that the maximum likelihood method can be depends on the estimating of the model parameters, as shown in the following figures.

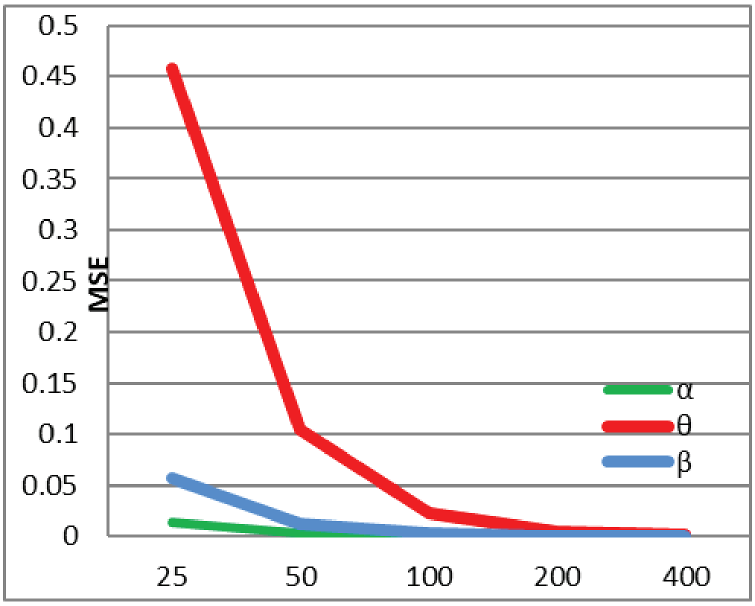

Fig. 2. Shows the change in(MSE) values when(β=0.5,α=0.9,θ=0.5).

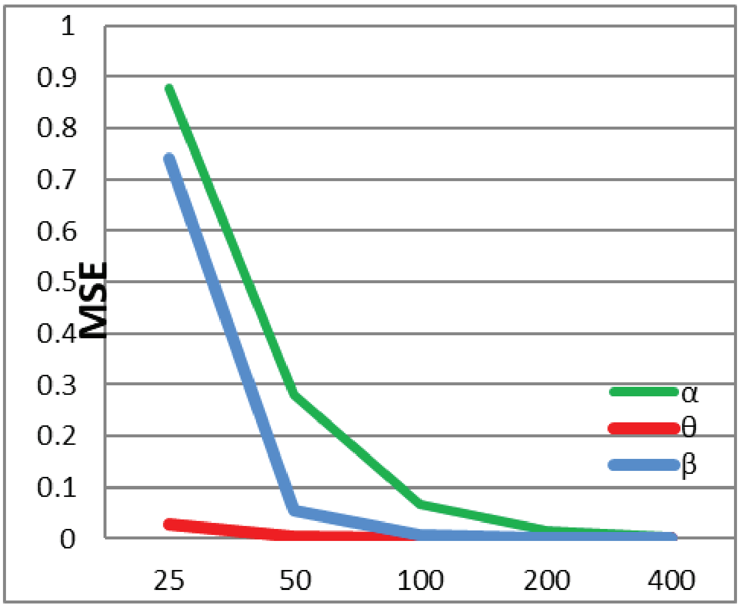

Fig. 3. Shows the change in(MSE) values when (β=0.9,θ=0.5,α=0.5).

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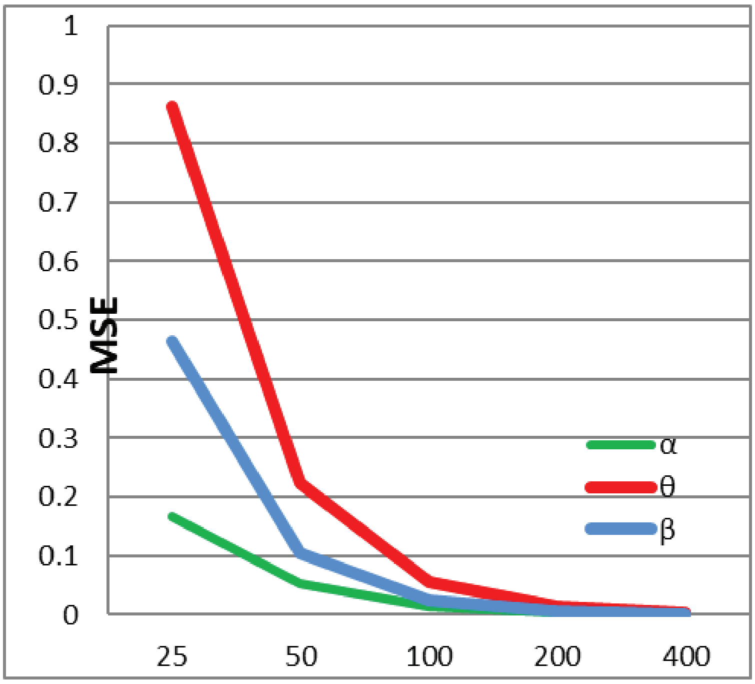

Fig. 4. Shows the change in (MSE) values when(β=0.5,α=0.5,θ=0.5).

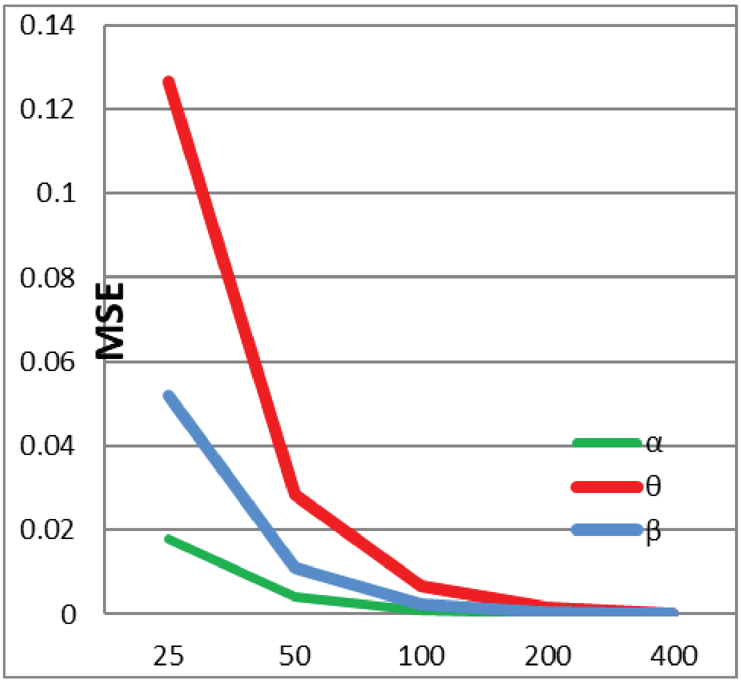

Fig. 5. Shows the change in (MSE) values when (β=2.5,θ=2,α=1.5).

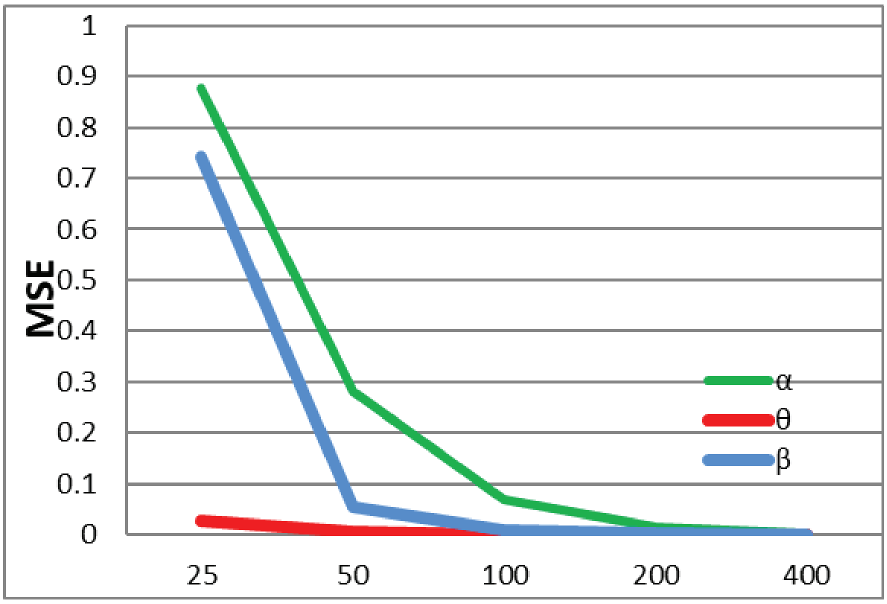

Fig. 6. Shows the change in (MSE) values when ($\beta = 1.5, \alpha = 2.5, \theta = 3.5$)

7 Conclusions

Under step (PALTs), it has estimated the distribution parameters of the "TMR" system, with each component lifetime. In addition, it has been computed the "RF, HRF, MRL" of the "TMR" system. In this study the MLE method has been taken. The determination of the optimal stress alteration time has been achieved by utilizing "two" deferent distinction criteria. The simulation experiments were used to estimate the mean square error (MSE) for the maximum likelihood method (MLE), and the result was shown that the mean square error values were relatively small.

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