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Soft Generalized αii-Closed Sets in Soft Topological Spaces

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Abstract . This work presented a novel kind of soft generalized closed sets and soft generalized open sets, which we will call soft generalized αii-closed sets and soft generalized αii-open sets in right now paper. The relationships between these two families and several other forms of soft sets, like soft generalized closed, soft generalized ii-closed, soft generalized α-closed, and soft semi-generalized closed sets, are studied and clarified through proofs and evidences such as such as, Theorem8 (If $(T,Z) \tilde{\subseteq} (W,Z) \tilde{\subseteq} (X,\eta,Z)$ is such that (T,Z) is a sGαICs in (X,η,Z), then (T,Z) is sGαIICs in relation to (W,η_W,Z)), Theorem9 (If $(T_1,Z),(T_2,Z)$ are sGαIICs, then so is their intersection) and Theorem10 (Each sGαIICs is a sGIICs).

1. Introduction

Mohammed A.A. & Abdullah B.S. introduced inter-open & ii –open set types during 2019[1] . Molodtsov along with many other experts presented the idea of soft sets and their characteristics in 1999, 2011 and 2015 [2,3,4]. Chen, B. & Kannan, K. defined "soft semi-open sets" and "soft -open sets" separately in the context of soft topological spaces in (2012, 2013) [5,6]. Askandar, S. W. & Mohammed, A. A. introduced the concept of soft i-open sets, soft ii-open sets, and soft ii–continuity in 2020 and 2022 [7, 8], which will be used in this study.

(sTs) is denoted by soft topological space (X,η,Z) in this article. Additionally, we indicate the soft set using the symbol (sS) and, in turn, the soft interior, soft closure, $Int(T,Z)$, $Cl(T,Z)$, and (T,Z) of the (sS). We refer to the η elements as soft open sets (sSs) . Additionally, their complements are referred to as closed soft sets, (sCs) .$\emptyset_z$, X_z Indicate the soft void and soft absolute sets privately.

We give out the fundamental theories of soft sets and soft topological spaces in Section 1. Furthermore, the basic definitions of a number of soft generalized closed, soft i-open, soft i-closed, soft ii-open, and soft ii-closed sets are provided. We define new concepts of soft generalized closed sets as soft generalized αii-closed sets in the second segment and discuss some of its interesting applications. The third section describes soft generalized αii-open sets and yields several important conclusions.

1.1 Preliminaries

Definition1. [2] X will be considered as the initial universe, where $P(X)$ is the power set of X , L is a set of parameters, also ($\emptyset \neq A \subseteq L$). A pair (Q,L) is a soft set over X where Q is defined as mapping $Q: A \to P(X)$. A soft se.t is a parameterized collecting with X subsections. Since $Q(l)$ in Particular $l \in A$ is seen as a soft set (Q,A) regardless of whether $l \notin A$, $Q(l) = \emptyset$, or $Q_A = \{Q(l): l \in A \subseteq L, Q: A \to P(X)\}$. All of these soft sets over X are together referred to as $SS(X_A)$.

Definition2. [3] If η is the set of all soft sets over X , then η is the soft topology on X if and only if:

a. $\emptyset_Z, X_Z$ are members of η .

b. the federation of any number of soft sets in η belong to η .

c. Finite number of soft sets intersecting in η belong to η.

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The soft topological space alternatively referred to as the triple (X, η, Z). Members of η (soft open sets) are named. They refer to their complements as (soft closed sets).

Proposition1. Consider $(T1, Z), (T2, Z)$ belongs to $SS(X_Z)$ then:

a. $((T1, Z)\ \tilde{\cup}\ (T2, Z))^C = (T1, Z)^C\ \tilde{\cap} (T2, Z)^C$.
b. $((T1, Z)\ \tilde{\cap} (T2, Z))^C = (T1, Z)^C\ \tilde{\cup}\ (T2, Z)^C$.[3].

Theorem1. Determine that (T, Z) is a soft set in (X, η, Z) . Then, there is:

a. $Int(T, Z)^C = (Cl(T, Z))^c$.
b. $Int(T, Z) = (Cl(T, Z)^c)^c$.
c. $Cl(T, Z)^c = (Int(T, Z))^c$ [9].

Definition3. Let (T, Z) be a soft set (sS) in (X, η, Z), (T, Z) is named:

i. "Soft i-open set "(sIOs), given that sOs $(Q, Z) \neq \emptyset, X$ exists and $(T, Z) \tilde{\subseteq} Cl((T, Z)\ \tilde{\cap}\ (Q, Z))$ [7].
ii. "Soft semi-open set", (sSOs) if : a. $(T, Z) \tilde{\subseteq} Cl(Int(T, Z))$. b. In the event that an sOs, $(Q, Z) \neq \emptyset, X$ occurs in which $(Q, Z) \tilde{\subseteq} (T, Z) \tilde{\subseteq} Cl(Q, Z)$, [5].
iii. "Soft α-open set", (sαOs) if " $(T, Z) \tilde{\subseteq} Int(Cl(Int(T, Z)))$ [6].
iv. "Soft int-open set", (sINTOs) if a sOs $(Q, Z) \neq \emptyset, X$ exists, such that, $Int(T, Z) = (Q, Z)$, [7].
v. "Soft ii-open set", (sIIOs) if a (sOs) $(Q, Z) \neq \emptyset, X$ exists, such that, $(T, Z)$is both soft i-open and soft inter-open set[7] .

A soft i-interior (also a soft α-interior, soft Semi-interior, soft inter-interior, and soft ii-interior) of a soft set (T, Z) is the federation of all *sIOs* (separately, (sαOs), (sSOs), (sINTOs), and (sIIOs)) over X included in(T, Z), denoted by $IInt(T, Z)$ (separately, $\alpha Int(T, Z)$, $SInt(T, Z)$, $INTInt(T, Z)$, and $IIInt(T, Z)$).

Soft i-closed (sICs) (separately, soft α-closed(sαCs), soft semi-closed (sSOs), soft inter-closed (sINTCs), and soft ii-closed set (sIICs)) is the whole complement of sIOs (separately, sαOs, sSOs, sINTOs, and (sIIOs)).

The soft i-closure (also the soft α-closure, soft semi-closure, soft inter-closure, and soft ii-closure) of (T, Z) is indicated by $ICl(T, Z)$ (separately, $\alpha Cl(T, Z)$, $SCl(T, Z)$, $INTCl(T, Z)$, and $IICl(T, Z)$) is the intersection of all *sICs* (separately, sαCs, sSCs, sINTCs, and sIICs) over X comprising (T, Z).

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Definition4. Soft generalized closed set, (sGCs) if $Cl(T, Z) \tilde{\subseteq} (Q, Z)$ wherein, $(T, Z) \tilde{\subseteq} (Q, Z)$ and (Q, Z) is a *sOs* in (X, η, Z) is taken into consideration by A (sS), (T, Z) in(X, η, Z).The term "soft generalized open set" (sGOs) refers to sGOs′S complement[10].

Definition5. (T, Z) in (X, η, Z) is called:

1. A soft generalized α-closed set (sGαCs) if $Cl(T, Z) \tilde{\subseteq} (Q, Z)$ is so that $(T, Z) \tilde{\subseteq} (Q, Z)$ and (Q, Z) together form an sαOs in (X, η, Z) [11].
2. The soft $\alpha -$ generalized closed set, (sαGCs), if $\alpha Cl(T, Z) \tilde{\subseteq} (Q, Z)$ is so that $(T, Z) \tilde{\subseteq} (Q, Z)$ and (Q, Z) together form an sOs in (X, η, Z) [12].
3. "Soft s*g-closed set", (sS^*GCs) if $Cl(T, Z) \tilde{\subseteq} (Q, Z)$ is so that $(T, Z) \tilde{\subseteq} (Q, Z)$ and (Q, Z) together form an sSOs in (X, η, Z) [13].
4. The soft semi-generalized closed set (sSGCs) is defined as follows: $SCl(T, Z) \tilde{\subseteq} (Q, Z)$ where $(T, Z) \tilde{\subseteq} (Q, Z)$ and (Q, Z) are an sSOs in (X, η, Z) [14].

The complement of (sGαCs) (resp., (sGαCs), sS^*GCs and(sSGCs)) is known as soft generalized α-open (sGαOs) (resp., soft α generalized open\ sαGOs, soft s*g open sS^*GOs and soft semi generalized open sSGCs.

Each sGαCs (resp., sαGCs, sS^*GCs, sSGCs and sGSCs in (X,η,Z) acquired $sG\alpha Cs(X_E)$) (resp., $s\alpha GCs(X_E)$, $sS^*GCs(X_E)$, $sSGCs(X_E)$ and $sGSCs(X_E)$).

Definition6. Consider (T,Z) be an sS in (X,η,Z). Pick" $\eta_{(T,Z)} = \{(G,Z) \tilde{\cap} (T,Z) : (G,Z) \in \eta\}$" the intellect of "soft topology" over (T,Z). The soft topology is named prorated soft topology of η over (T,Z) also $\{(T,Z), \eta_{(T,Z)}\}$ is defined as (X,η,Z) soft subspace[15].

Theorem2. Each sOs is a sIOs [7].

Proposition2. Each sCs is a sICs [7].

Theorem3. Each sSOs is a sIIOs [7].

Propositin3. Each sSCs is a sIICs.[16]

Theorem4. Each sαOs is a sSOs [7].

Proposition4. Each sαCs is a sSCs. [16]

Theorem5. Each sαOs is a sIIOs [7].

Propostion5. Each sαCs is a sIICs [16].

1.2 Soft Generalized αii − Closed Sets

Definition7. Let (T,Z) be a (sS) in (X,η,Z), then (T,Z) considers:

1. Soft generalized αii-closed set, If $IICl(T,Z) \tilde{\subseteq} (Q,Z)$ is such that $(T,Z) \tilde{\subseteq} (Q,Z)$ and (Q,Z) is a *sαOs* in (X,η,Z). $sG\alpha IICs(X_Z)$ represents all of the sGαIICs in the group.
2. Soft generalized ii-closed set, (sGIICs) since $IICl(T,Z) \tilde{\subseteq} (Q,Z)$ " wherein $(T,Z) \tilde{\subseteq} (Q,Z)$ and (Q,Z) is a *sOs* in (X,η,Z). The group of all *sGIICs* is designated by $sGIICs(X_Z)$.

Theorem6. Each sIICs is a sGαIICs.

Proof: Assume that a sIICs, (T,Z) in (X,η,Z) and (Q,Z) is a *sαOs* wherein, $(T,Z) \tilde{\subseteq} (Q,Z)$, we get, $IICl(T,Z) = (T,Z) \tilde{\subseteq} (Q,Z)$. Henceforth, (T,Z) is a sGαIICs■

Theorem7. If $(T,Z) \tilde{\subseteq} (L,Z) \tilde{\subseteq} IICl(T,Z)$, wherein, (T,Z) is a sGαIICs, then so is (L,Z).

Proof: Consider $(L,Z) \tilde{\subseteq} (Q,Z)$ and (Q,Z) be a sαOs in (X,η,Z), then $(T,Z) \tilde{\subseteq} (Q,Z)$. Since (T,Z) is a sGαIICs, $(L,Z) \tilde{\subseteq} IICl(T,Z)$, we conclude that "$IICl(L,Z) \tilde{\subseteq} IICl(T,Z)$". We get " $IICl(L,Z) \tilde{\subseteq} IICl(T,Z) \tilde{\subseteq} (Q,Z)$". We obtained that " $IICl(L,Z) \tilde{\subseteq} (Q,Z)$". Henceforth, (L,Z) is a sGαIICs■

Theorem8. If $(T,Z) \tilde{\subseteq} (W,Z) \tilde{\subseteq} (X,\eta,Z)$ is such that (T,Z) is a sGαICs in (X,η,Z) , then (T,Z) is sGαIICs in relation to (W,η_W,Z).

Proof: Assume $(T,Z) \tilde{\subseteq} (Q,Z)$ and (Q,Z) represent a sαOs in (W,Z). Since $(T,Z) \tilde{\subseteq} (W,Z)$, gives us $(T,Z) \tilde{\subseteq} (Q,Z) \tilde{\cap} (W,Z)$, $(Q,Z)$is a sαOs in (W,η_W,Z). Consequently, there is an sαOs (M,Z) in X where $(Q,Z) = (M,Z) \tilde{\cap} (W,Z)$, then, $(T,Z) \tilde{\subseteq} (Q,Z) \tilde{\subseteq} (M,Z)$ with (T,Z) representing a sGαIICs in (X,η,Z) gives us $IICl(T,Z) \tilde{\subseteq} (M,Z)$ by $IICl(T,Z) \tilde{\cap} (W,Z)$ is a soft ii-closure of (T,Z) in (W,η_W,Z), giving us $IICl(T,Z) \tilde{\cap} (W,Z) \tilde{\subseteq} (Q,Z)$. From this point on, (T,Z) is a sGαIICs with respect to (W,η_W,Z)■

Theorem9. If $(T_1,Z),(T_2,Z)$ are sGαIICs, then so is their intersection.

Proof: Consider $(T_1,Z),(T_2,Z)$ as a sGαIICs in (X,η,Z) and $(Q_1,Z),(Q_2,Z)$ are any sαOs where in $(T_1,Z)\tilde{\subseteq}(Q_1,Z)$ and $(T_2,Z)\tilde{\subseteq}(Q_2,Z)$, then, $IICl(T_1,Z)\tilde{\subseteq}(Q_1,Z)$ and $IICl(T_2,Z)\tilde{\subseteq}(Q_2,Z)$ $(IICl((T_1,Z)\tilde{\cap}(T_2,Z))\tilde{\subseteq}IICl((T_1,Z)\tilde{\cap}IICl(T_2,Z))$, we get : $Cl((T_1,Z)\tilde{\cap}(T_2,Z))\tilde{\subseteq}((Q_1,Z)\tilde{\cap}I(Q_2,Z))$ and since the intersection between any two sαOs is a sαOs, put $\left((Q_1,Z)\tilde{\cap}I(Q_2,Z)\right)=(Q,Z)$, with (Q,Z) is a sαOs, $IICl((T_1,Z)\tilde{\cap}(T_2,Z))\tilde{\subseteq}(Q,Z)$. Henceforth, $(T_1,Z)\tilde{\cap}(T_2,Z)$ is a sGαIICs■

Theorem10. Each sGαIICs is a sGIICs.

Proof: Consider (T_1,Z) is a sGαIICs in (X,η,Z) and (Q,Z) is any sOs wherein, $(T,Z)\tilde{\subseteq}(Q,Z)$, then $IICl(T,Z)\tilde{\subseteq}(Q,Z)$ since each sOs is sαOs with (Q,Z) is a sOs . Henceforth, (T,Z) is a sGIICs■

Theorem11. Each sGαCs *sGαCs* is a sGαIICs.

Proof: Assume that (T,Z) is a sGαCs in (X,η,Z) and (Q,Z) is any sαOs wherein, $(T,Z)\tilde{\subseteq}(Q,Z)$, $IICl(T,Z)\tilde{\subseteq}Cl(T,Z)\tilde{\subseteq}(Q,Z)$ will that mean $IICl(T,Z)\tilde{\subseteq}(Q,Z)$ with (O,E) is a sαOs . Henceforth, (T,Z) is a sGαIICs■

Theorem12. Each sSGCs *sSGCs* is a sGαCIIs.

Proof: Consider (T,Z) is an sSGCs in (X,η,Z) and $(T,Z)\tilde{\subseteq}(Q,Z)$ where (Q,Z) is any sαOs and sSOs, since, each sSCs is a sIICs "(Proposition3)", then $IICl(T,Z)\tilde{\subseteq}SCl(T,Z)\tilde{\subseteq}(Q,Z)$. Henceforth, (T,Z) is a sGαIICs■

1.3 Soft Generalized *αii* -Open Sets

Definition8. Let (T,Z) be a (sS) in (X,η,Z), then (T,Z) considers as a soft generalized αii-open set, (sGαIIOs) when its complement $(T,Z)^C$ is a sGαIICs. The group of every sGαIIOs is denoted by sGαIIOs(X_Z).

Theorem13. A soft set (T,Z) in (X,η,Z), is sGαIIOs when and if only $(U,Z)\tilde{\subseteq}IIInt(T,Z)$ where (U,Z) is a sαCs within $(U,Z)\tilde{\subseteq}(T,Z)$.

Proof: Assume that (T,Z) be sGαIIOs with $(U,Z)\tilde{\subseteq}(T,Z)$ and (U,Z) is a sαCs. Then$(T,Z)^C$ is sGαIICs and $(U,Z)^C$ is sαOs with, $(T,Z)^C\tilde{\subseteq}(U,Z)$, $IICl(T,Z)^C\tilde{\subseteq}(U,Z)^C$. Henceforth,$(U,Z)\tilde{\subseteq}IIInt(T,Z)$.

Assume, in reverse, that $(U,Z)\tilde{\subseteq}IIInt(T,Z)$ with $(U,Z)\tilde{\subseteq}(T,Z)$ and $(U,Z)$is sαCs. Think of (M,Z) as a sαOs that contains $(T,Z)^C$.

Consequently, $(M,Z)^C\tilde{\subseteq}IIInt(T,Z)$, then, $IICl(T,Z)^C\tilde{\subseteq}(M,Z)$. Henceforth, $(T,Z)^C$ is sGαIICs. Which implies (T,Z) be sGαIIOs■

Theorem14. If $IIInt(T,Z)\tilde{\subseteq}(L,Z)\tilde{\subseteq}(T,Z)$, wherein, (T,Z) is a sGαIIOs, then so is(L,Z).

Proof: Consider $IIInt(T,Z)\tilde{\subseteq}(L,Z)\tilde{\subseteq}(T,Z)$, $(L,Z)\tilde{\subseteq}(Q,Z)$ then,

$(T,Z)^C\tilde{\subseteq}(L,Z)^C\tilde{\subseteq}IICl((T,Z)^C)$ with $(T,Z)^C$ is sGαIICs. Thus, $(L,Z)^C$ is sGαIICs "(Theorem7)". Henceforth, (L,Z) is sGαIIOs■

Corollary1. Each sGαIIOs is a sGIIOs.

Proof: If we take (T,Z) to be a sGαIIOs in (X,η,Z), we obtain $(T,Z)^C$ to be an *sGαIICs* , which suggests $(T,Z)^C$ to be sGIICs "(Theorem 10)." From now on, $(T,Z)^C$ is a sGIIOs■

Corollary2. Each sGαOs is a sGαIIOs.

Proof: Let's say that (T, Z) is a sGαOs in (X, η, Z). Then, $(T, Z)^C$ is an sGαCs, and $(T, Z)^C$ is sGαIICs "(Theorem11)". Henceforth, (T, Z) is a sGαIIOs■

Corollary3. Each sSGOs is a sGαIIOs.

Proof: If we assume that (T, Z) is a sSGOs in (X, η, Z), Then, $(T, Z)^C$ is an sSGCs , we conclude that $(T, Z)^C$ is sGαIICs. "(Theorem12)". Henceforth, (T, Z) is a sGαIIOs■

Corollary4. If $(T_1, Z), (T_2, Z)$ are sGαIIOs, then so is their union.

Proof: Take note of $(T_1, Z), (T_2, Z)$ as a sGαIIOs in (X, η, Z). We obtain, $(T_1, Z)^C, (T_2, Z)^C$ are sGαIIOs. "(Theorem9)" gives us the result that $(T_1, Z)^C \tilde{\cap} (T_2, Z)^C = ((T_1, Z) \tilde{\cup} (T_2, Z))^C$ is a sGαIICs. Therefore, $(T_1, Z) \tilde{\cup} (T_2, Z)$ is now a sGαIIOs■

Corollary5. Each sIIOs is a sGαIIOs.

Proof: Assume that a sIIOs, (T, Z) in (X, η, Z) , We obtain, $(T, Z)^C$ is sGIICs, by "(Theorem6)" we get, $(T, Z)^C$ is sGαIICs. Henceforth, (T, Z) is a sGαIIOs■

3. Conclusions

According to the above we concluded that The relationship between sαOs, sIIOs, sSOs, and sOs determines the relationship between sGαIICs, sGαCs, sGCs, sGIICs and sSGCs**,** Each sIICs is a sGαIICs, If $(T_1, Z), (T_2, Z)$ are sGαIICs, hence, their junction has to be sGαIICs, Each sGαIICs is a sGIICs**,** Each sGαCs is a sGαIICs**,** Each sSGCs is a sGαIICs**,** Each is a sGIIOs**,** All sGαOs is a sGαIIOs and If $(T_1, Z), (T_2, Z)$ are sGαIIOs, then their federation has to also be sGαIIOs.

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