Long-term filtration of particles in a porous medium

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Abstract: The formation of grout sediment in the pores of loose rock increases the water resistance of the soil and strengthens the foundation. A one-dimensional model of filtration in a porous medium considers the particles transport by the flow of a carrier fluid and the deposition of particles on the framework of a porous medium. The purpose of the work is to study the concentrations of suspended and settled particles of a suspension over a long time. Exact and asymptotic methods are used to obtain a solution to the model. The exact solution is presented in an implicit integral form. A set of solutions in the form of traveling waves with an arbitrary initial condition and their asymptotics are constructed. For the exact solution, an explicit second-order asymptotic solution for a long time is obtained as an expansion in decreasing exponents. Comparison of the asymptotic solution with the traveling waves makes it possible to choose a single traveling wave corresponding to the exact solution. The closeness of the traveling wave to the exact solution of the filtration model is verified numerically. The traveling wave found determines the explicit asymptotics of the concentration of deposited particles for a long time.

Keywords. Deep bed filtration, porous medium, exact solution, asymptotics, traveling wave solution

1 Introduction

Particles transport and filtration in porous media is actively used in construction technologies. The spread of solid particles of liquid grout in porous soil makes it possible to strengthen the foundation and create water resistant walls in the soil for underground structures [1-7].

During the filtration, particles are transferred and deposited on the framework of a porous medium. Deep bed filtration assumes that particles penetrate the entire depth of the porous medium [8-11]. Various forces act on particles carried by a fluid flow: hydrodynamic, electrostatic, mechanical, etc. [12]. Particles can be deposited on the walls of wide pores, get stuck at the entrance of narrow pores, or enter dead-end pores. Under real conditions, depending on the physical, chemical, and geometry of the porous medium and particles, a certain retention mechanism becomes predominant. If the transverse dimensions of the pores

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and the particle diameters are of the same order, then the main capture mechanism is the size exclusion mechanism, in which the particles are transported through wide pores and are blocked in the narrow pores. The deposited particles are motionless [15 – 17].

Basic deep bed filtration model determines the volumetric concentrations of suspended and retained particles. The sediment growth is determined by the filtration function, which depends on the current concentration of the retained particles. The simplest constant filtration function models filtration only over a short period of time. As the sediment grows, the number of vacancies for the retained particles on the porous medium framework decreases and the filtration process slows down. The decreasing filtration function allows one to model filtration over a large time interval. After a long period of filtration, the retained particles concentration stabilizes and filtration stops: all particles injected into the porous medium leave it at the exit without sediment formation. This phenomenon is described by a filtration blocking function that vanishes when the sediment limit value is reached. The most commonly used linear filtration function is called the Langmuir coefficient [18]. However, experimental studies show that the filtration function can be more complex.

In this paper, we construct the asymptotics of the filtration problem for a large time for filtration function of a general form. It is shown that the sediment concentration near the limit value is determined not only by the local behavior of the filtration function near the root, but also by the integral over the entire domain of its definition. The solution is found in the form of traveling waves. The obtained asymptotics allows us to determine the only traveling wave corresponding to the exact solution of the filtration problem.

2 Mathematical model

In the domain $\Omega = \{0 \le x \le 1,\ t \ge 0\}$ 1-D model of deep bed filtration determines the concentrations of suspended particles $C(x,t)$ and retained particles $S(x,t)$. The dimensionless system has the form [19]

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial S}{\partial t} = 0 \,, \tag{1}$$

$$\frac{\partial S}{\partial t} = \Lambda(S) C \,, \tag{2}$$

where the blocking filtration function $\Lambda(S)$ is non-negative, decreases and vanishes at the point $S = S_m$.

For the uniqueness of the solution the initial and boundary conditions are set. They correspond to the injection of a suspension or colloid of constant concentration into a porous medium filled with pure water without suspended and retained particles:

$$C(x,t)\big|_{x=0} = 1 \,, \tag{3}$$

$$C(x,t)\big|_{t=0} = 0, \quad S(x,t)\big|_{t=0} = 0 \,. \tag{4}$$

From the inconsistency of conditions (3) and (4), it follows that the solution $C(x,t)$ has a gap at the concentration front $t = x$. Before the concentration front, the solution is zero, and behind the front, it is positive.

At $t \ge x$ the solution to problem (1)-(4) is obtained in implicit form [20, 21]

$$\int_0^{S_0} \frac{ds}{\Lambda(s)} = t \,, \tag{5}$$

$$\int_{S_0(t-x)}^{S(x,t)} \frac{ds}{s\Lambda(s)} = -x \,. \tag{6}$$

Here $S_0(t)$ is the concentration of retained particles at the entry of the porous medium $x=0$.

For known $S(x,t)$, the solution $C(x,t)$ can be determined from Eq. (2), or one can use the Riemann invariant, which relates the solutions on the characteristics of the system [22, 23]:

$$C(x,t) = \frac{S(x,t)}{S_0(t-x)} \,. \tag{7}$$

To determine the behavior of solutions for long time, consider a traveling wave on the plane (x,t) [24]

$$C = C(w), \quad S = S(w), \quad w = x - ut \,, \tag{8}$$

which moves at a constant speed u. System (1), (2) is supplemented by the conditions at infinity:

$$w = -\infty: \; C = 1, \; S = S_m, \quad w = +\infty: \; C = 0, \; S = 0 \,. \tag{9}$$

Equation (1) takes the form

$$-uC' + C' - uS' = 0 \,. \tag{10}$$

Integration of equation (10) gives

$$(1-u)C - uS = K, \quad K = const \,. \tag{11}$$

It follows from conditions (9) that $K=0$ and

$$u = \frac{1}{1+S_m}, \quad C = \frac{S}{S_m} \,. \tag{12}$$

Substitute formulae (8), (12) into equation (2):

$$\frac{S'}{1+S_m} + \Lambda(S)\frac{S}{S_m} = 0 \,. \tag{13}$$

For the uniqueness of the solution, the condition at the point $w=0$ can be set

$$S\big|_{w=0} = S^*, \quad 0 < S^* < S_m \,. \tag{14}$$

Solution of problem (13), (14) has the form

$$\int_{S^*}^{S} \frac{ds}{s\Lambda(s)} = -\frac{1+S_m}{S_m} w \,. \tag{15}$$

Comparison of formula (15) with solution (6) shows that, at a certain value S^*, the traveling wave is close to the exact solution.

Fig. 1 demonstrates traveling waves for problem (1)-(4) with filtration function $\Lambda(S)=1-S^2$ for various $S*$.

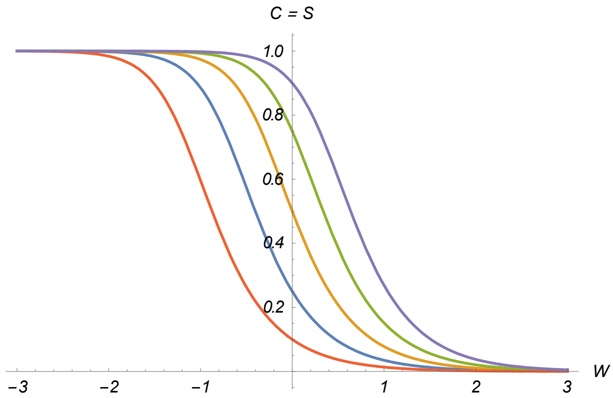

Fig. 1. Traveling waves solutions for $S*=0.1;\ 0.25;\ 0.5;\ 0.75;\ 0.9$.

3 Asymptotic solution for a long time

Let, in a neighborhood of the root $S=S_m$, the filtration function admits the representation

$$\Lambda(S)=\lambda_1(S-S_m)+0.5\lambda_2(S-S_m)^2+\dots,\quad \lambda_1=\Lambda'(S_m)<0,\quad \lambda_2=\Lambda''(S_m)\,. \qquad (16)$$

To find the asymptotics [25, 26], we will seek a solution to problem (1)-(4) as a series in decreasing exponents:

$$S(x,t)=S_m+s_1(x)e^{\lambda_1 t}+s_2(x)e^{2\lambda_1 t}+O\left(e^{3\lambda_1 t}\right), \qquad (17)$$

$$C(x,t)=1+c_1(x)e^{\lambda_1 t}+c_2(x)e^{2\lambda_1 t}+O\left(e^{3\lambda_1 t}\right). \qquad (18)$$

Let us represent the integral on the left side of Eq. (5) as

$$\begin{aligned}\int_0^{S_0}\frac{ds}{\Lambda(s)}&=\int_0^{S_0}\left(\frac{1}{\Lambda(s)}-\frac{1}{\lambda_1(s-S_m)}\right)ds+\int_0^{S_0}\frac{ds}{\lambda_1(s-S_m)}=\\&=I-\int_{S_0}^{S_m}\left(\frac{1}{\Lambda(s)}-\frac{1}{\lambda_1(s-S_m)}\right)ds+\frac{1}{\lambda_1}\ln\frac{S_m-S_0}{S_m}.\end{aligned} \qquad (19)$$

Here

$$I=\int_0^{S_m}\left(\frac{1}{\Lambda(s)}-\frac{1}{\lambda_1(s-S_m)}\right)ds\,.$$

Substitute expansion (16) into the integral on the left side of (19). Equation (5) takes the form

$$I+\frac{\lambda_2}{2\lambda_1^2}(S_m-S_0)+O(S_m-S_0)^2+\frac{1}{\lambda_1}\ln\frac{S_m-S_0}{S_m}=t\,. \qquad (20)$$

Solving equation (20), we find the asymptotics in the form (17) for the function $S_0(t)=S(0,t)$:

$$S_0(t)=S_m\left(1-e^{\lambda_1(t-I)}+\frac{\lambda_2}{2\lambda_1}S_m e^{2\lambda_1(t-I)}\right)+O\left(e^{3\lambda_1 t}\right). \tag{21}$$

Similarly to formula (19), the integral (6) can be written in the form

$$\begin{aligned}\int_{S_0(t-x)}^{S(x,t)}\frac{ds}{s\Lambda(s)}&=\int_{S_0}^{S}\left(\frac{1}{\Lambda(s)}-\frac{1}{\lambda_1(s-S_m)}\right)\frac{ds}{s}+\int_{S_0}^{S}\frac{ds}{\lambda_1 s(s-S_m)}=\\&=-\frac{\lambda_2}{2\lambda_1^2}\left(S-S_0-S_m\ln\frac{S}{S_0}\right)+\frac{1}{\lambda_1 S_m}\ln\frac{\left(S_m-S\right)S_0}{S\left(S_m-S_0\right)}+O(S-S_0)^2.\end{aligned} \tag{22}$$

Substitute expansions (21), (22) into equation (6) and obtain the asymptotics of the deposit concentration

$$S=S_m\left(1-e^{-\lambda_1 S_m x}e^{\lambda_1(t-x-I)}+\left(-e^{-\lambda_1 S_m x}+(1+\frac{\lambda_2}{2\lambda_1}S_m)e^{-2\lambda_1 S_m x}\right)e^{2\lambda_1(t-x-I)}\right)+O\left(e^{3\lambda_1 t}\right). \tag{23}$$

To find the asymptotic solution $C(x,t)$, substitute expansions (21) and (23) into the Riemann invariant (7):

$$\begin{aligned}C&=1+\left(1-e^{-\lambda_1 S_m x}\right)e^{\lambda_1(t-x-I)}+\\&+\left(1-\frac{\lambda_2}{2\lambda_1}S_m-2e^{-\lambda_1 S_m x}+(1+\frac{\lambda_2}{2\lambda_1}S_m)e^{-2\lambda_1 S_m x}\right)e^{2\lambda_1(t-x-I)}+O\left(e^{3\lambda_1 t}\right).\end{aligned} \tag{24}$$

Let us find the asymptotics of the traveling wave (15) for a long time, i.e. at $w\to-\infty$. Similarly to expansions (19) and (22), represent the integral on the left side of formula (15) in the form

$$\begin{aligned}\int_{S*}^{S}\frac{ds}{s\Lambda(s)}&=\int_{S*}^{S}\left(\frac{1}{\Lambda(s)}-\frac{1}{\lambda_1(s-S_m)}\right)\frac{ds}{s}+\int_{S*}^{S}\frac{ds}{\lambda_1 s(s-S_m)}=\\&=I*-\int_{S}^{S_m}\left(\frac{1}{\Lambda(s)}-\frac{1}{\lambda_1(s-S_m)}\right)\frac{ds}{s}+\frac{1}{\lambda_1 S_m}\ln\frac{\left(S_m-S\right)S*}{S\left(S_m-S*\right)}=\\&=I*-\frac{\lambda_2}{2\lambda_1^2}\left(S_m-S-S_m\ln\frac{S_m}{S}\right)+\frac{1}{\lambda_1 S_m}\ln\frac{\left(S_m-S\right)S*}{S\left(S_m-S*\right)}+O(S_m-S)^2,\end{aligned} \tag{25}$$

where

$$I*=\int_{S*}^{S_m}\left(\frac{1}{\Lambda(s)}-\frac{1}{\lambda_1(s-S_m)}\right)\frac{ds}{s}.$$

Substitute expansion (25) into (15) and obtain the asymptotics of the traveling wave solution:

$$S=S_m\left(1-\frac{S_m-S*}{S*}e^{-\lambda_1(1+S_m)w-\lambda_1 S_m I*}\right)+O\left(e^{-2\lambda_1(1+S_m)w}\right). \tag{26}$$

The main exponents in formula (23) and (26) coincide. Equate the coefficients for decreasing exponents:

$$\frac{S_m - S*}{S*} e^{-\lambda_1 S_m I*} = e^{-\lambda_1 I}. \tag{27}$$

Formula (27) specifies the initial value S_S^* for the traveling wave corresponding to the solution $S(x,t)$. Note that the following terms of the asymptotics of traveling waves and exact solutions are different.

The initial value S_C^* corresponding to the solution $C(x,t)$ depends on the variable x:

$$\frac{S_m - S*}{S*} e^{-\lambda_1 S_m I*} = \left(1 - e^{\lambda_1 S_m x}\right) e^{-\lambda_1 I}. \tag{28}$$

For each fixed x, one can determine S_C^* from equation (28)

4 Numerical results

Consider two examples of filtration problem with different filtration functions and compare the exact solution with its asymptotics and the appropriate traveling wave.

Example 1. Let the filtration function be the Langmuir coefficient

$$\Lambda(S) = 1 - S,$$

then $S_m = 1, \quad \lambda_1 = -1, \quad \lambda_2 = 0$, integrals $I = 0, \quad I* = 0$ and $S_S^* = 0.5, \quad S_C^*(x) = 1/(2 - e^{-x})$.
Asymptotics (23) takes the form

$$S = 1 - e^{-(t-2x)} + \left(1 - e^{-x}\right) e^{-2(t-2x)} + O\left(e^{-3t}\right),$$
$$C = 1 + \left(e^{-x} - 1\right) e^{-(t-2x)} + \left(e^{-2x} - 2e^{-x} + 1\right) e^{-2(t-2x)} + O\left(e^{-3t}\right).$$

Exact solution to problem (1)-(4) behind the front

$$S(x,t) = \frac{e^{t-x} - 1}{e^{t-x} + e^x - 1}, \qquad C(x,t) = \frac{e^{t-x}}{e^{t-x} + e^x - 1}.$$

Formula (15) takes the form

$$\int_{S*}^{S} \frac{ds}{s(1-s)} = -2w. \tag{29}$$

Solutions to equation (15)

$$S(w) = \frac{S*}{(1-S*)e^{2w} + S*}.$$

Substituting the initial conditions gives

$$S_{TWS}(w) = \frac{1}{e^{2w} + 1}, \qquad C_{TWS}(w) = \frac{1}{\left(1 - e^{-x}\right) e^{2w} + 1}.$$

Fig. 2 shows the exact solution, the asymptotics, and the traveling wave at the porous medium exit.

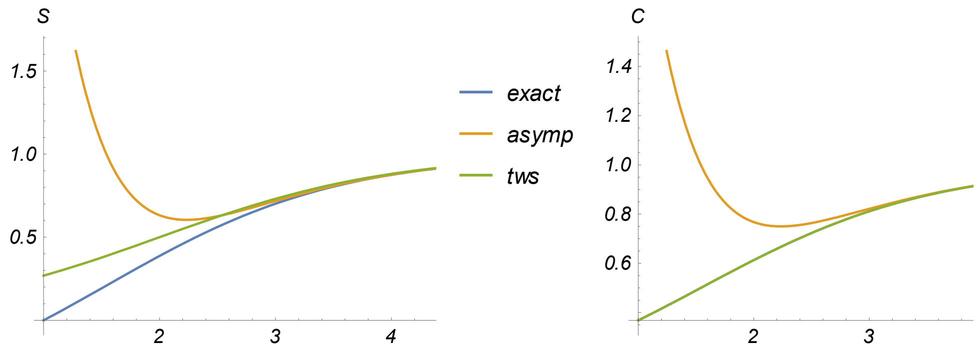

Fig. 2. Exact and asymptotic concentrations S and C at $x=1$ for the Langmuir coefficient.

For x=1, the exact solution C coincides with the selected traveling wave.

Example 2. Let

$$\Lambda(S)=1-S^2,$$

then $S_m=1,\ \lambda_1=-2,\ \lambda_2=-2$, integrals $I=0.5\ln 2,\ I^*=0.5\left(\ln 2-\ln(1+S^*)\right)$ and $S_S^*=\sqrt{2}-1,\ S_C^*(x)=0.445$.

Asymptotics (23) takes the form

$$S=1-2e^{-2(t-2x)}+4\left(1.5-e^{-2x}\right)e^{-4(t-2x)}+O\left(e^{-6t}\right),$$
$$C=1+2\left(e^{-2x}-1\right)e^{-2(t-2x)}+4\left(0.5e^{-4x}-2e^{-2x}+1.5\right)e^{-4(t-2x)}+O\left(e^{-6t}\right).$$

Exact solution to problem (1)-(4) is written in explicit form:

$$S(x,t)=\frac{e^{2(t-x)}-1}{\sqrt{\left(e^{2(t-x)}-1\right)^2+4e^{2t}}},\qquad C(x,t)=\frac{\left(e^{2t-2x}+1\right)\left(e^{2t}-1\right)}{\left(e^{2t-2x}-1\right)\sqrt{\left(e^{2t}-1\right)^2+4e^{2t+2x}}}.$$

Formula (15) takes the form

$$\int_{S^*}^{S}\frac{ds}{s(1-s^2)}=-2w. \tag{30}$$

Solution to equation (30)

$$S(w)=\frac{S^*}{\sqrt{(1-S^{*2})e^{2w}+S^{*2}}}.$$

Substitution of the initial conditions yields

$$S_{TWS}(w)=\frac{1}{\sqrt{2(\sqrt{2}+1)e^{4w}+1}},\qquad C_{TWS}(w)=\frac{1}{\sqrt{4.059e^{4w}+1}}.$$

Fig. 3 presents the exact solution, asymptotics and traveling wave separately for the concentrations S and C.

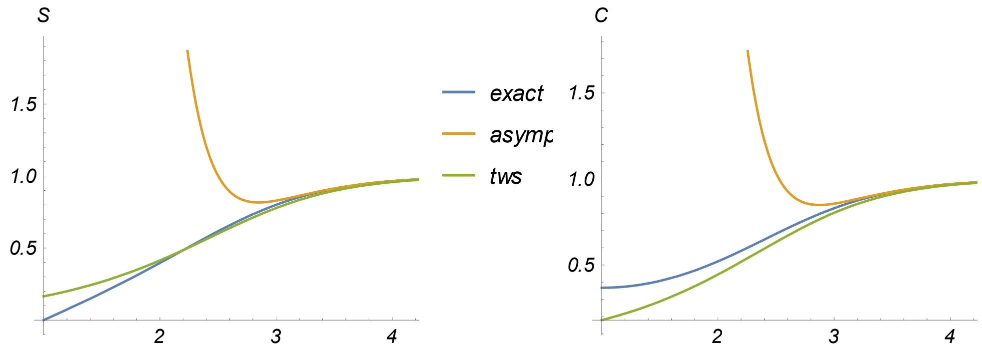

Fig. 3. Exact and asymptotic concentrations S and C at $x=1$ for the filtration function $\Lambda(S)=1-S^2$.

The selected traveling wave, the asymptotics, and the exact solution practically coincide at $t>3$.

5 Discussion

Some filtration models admit an exact solution. As a rule, the solution is obtained in a complex implicit form. To calculate the filtration problem or select parameters for fine-tuning experiments, it is convenient to use explicit analytical formula solutions [27, 28].

The asymptotics approximate the exact solutions well for a sufficiently long time, see Fig. 2 and 3. A comparison of the asymptotic solutions shows that, with an appropriate choice of the initial condition, the traveling wave approximates the solution with the same accuracy as the second order asymptotics. This means the importance of finding traveling waves for filtration problems.

A set of solutions in the form of a traveling wave can be obtained for systems of equations whose coefficients do not explicitly depend on time and spatial coordinates. The traveling wave is given implicitly, as well as the exact solution. To choose a specific traveling wave that approximates the solution of the filtration model, it is necessary to compare the asymptotics of the exact solution and the traveling wave.

The advantage of the asymptotic expansion is the explicit form of the analytic formula. To solve the inverse filtration problem by an asymptotic method, explicit formulae are required [29]. Thus, when solving the filtration problem, it is useful to study both traveling waves and the asymptotic solution.

6 Conclusions

Analytic modelling of long-term filtration of suspensions and colloids in a porous medium yields the following conclusions:

- The exact solution to the filtration problem is given in an implicit form.
- An asymptotic expansion of the exact solution up to the second order is obtained.
- A set of solutions in the form of a traveling wave is found.
- A single traveling wave is found that optimally approximates the exact solution for a long time.
- The found explicit analytical formulae make it possible to control the model parameters during experiments.

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