

Number of concrete strength tests using the elastic rebound method

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Abstract. The article discusses the determination of the required number of experiments when determining the compressive strength of concrete using the elastic rebound method. It is shown that it is not correct to accept a deterministic value for the minimum number of experiments for all grades (classes of concrete). This value depends on the average strength of concrete and the instrument error of the device used. For a given coefficient of variation, with increasing concrete strength, the required number of experiments increases. For the maximum permissible coefficient of variation of concrete strength of 13.5%, calculated dependences of the minimum number of experiments on the average strength of concrete were obtained for the minimum and maximum instrumental errors of the sclerometers used, determined by analyzing the passport data used in Russian organizations involved in the inspection of buildings and structures. The proposed method, which relates the strength of the material of the structure under study and the instrumental error of the device used, can be extended to other methods of determining strength by non-destructive methods and can be the basis of the technical specifications for the creation of devices for non-destructive testing of concrete.

Keywords: concrete strength, Schmidt hammer, number of experiments, instrument error, confidence interval, concrete grade, concrete class

INTRODUCTION

According to [1], one of the non-destructive methods for determining the strength of concrete is the elastic rebound method implemented in sclerometers (Schmidt hammers). The Schmidt hammer, also known as the Swiss hammer, is a device for measuring the elastic properties or strength of concrete and rocks, was invented by Ernst Heinrich Wilhelm Schmidt, a Swiss engineer in 1948 [2]. The hammer measures the rebound of the spring-loaded mass hitting the surface of the sample. Its return depends on the hardness of the concrete and is measured using test equipment. Using the conversion table, the rebound value can be used to determine the compressive strength of concrete. Measurements with Schmidt hammer are made on an arbitrary scale in the range from 10 to 100. Currently, three types of sclerometers are produced: mechanical, electronic and ultrasonic. The first two types perform measurement according to the shock-pulse method standardized by

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GOST. It consists in determining the length of the rebound of the built-in mechanism that transmits the impact of a solid surface.

A large number of works are devoted to the use of Schmidt hammers. The works [3, 4] examine the features of using the Schmidt hammer to determine the strength of rocks. In [5], a study is conducted on the combined use of acoustic methods and the elastic rebound method. In [6-9], attempts were made to relate the strength determined by the elastic rebound method with the elastic modulus using artificial neural networks or regression analysis.

In our country, the following models of Schmidt hammers are used: for concrete – models 225A, 225AII, 225II, 225II2-1; for bricks, stones and small concrete products with thin walls - models 75A, 75II; for mortar in brickwork - 20A, 20II models; for high-strength concrete - models 1000A, 550A, 450A, produced by Vostok-7; Schmidt hammers NOVOTEST MIII-225, MIII-75, MIII-20, differing in impact energy; device OMIII-1, produced by a number of domestic manufacturers and original hammers from Proceq: Original Schmidt, Silvers Schmidt, Original Schmidt, Live Schmidt OS.

The maximum permissible (normative) coefficient of variation in concrete strength is 0.135 (13.5%). GOST [1] regulates for the elastic rebound method the total number of tests is 9, the minimum distance between measurement points in a section is 30 mm, the minimum distance from the edge of the structure to the measurement point is 50mm, the minimum thickness of the structure is 100 mm. However, the authors were unable to find a statistical justification for the number of tests on a concrete section using the methods regulated in GOST; moreover, in [1] this value is deterministic and independent of the error of the device used and the grade of concrete. Which made it necessary to consider the influence of these factors on the results obtained.

MATERIALS AND METHODS

The article is of a computational and analytical nature. The possibility of using Schmidt hammers with a minimum and maximum passport measurement error at a normalized value of the coefficient of variation is considered. The approach of the classical error theory was applied, which allowed determining the minimum number of measurements, linking the predicted grade (concrete class) with the magnitude of the error of change and the coefficient of variation of concrete strength.

In a number of works, for example [10 - 12]. describes a technique according to which, to determine the number of measurements, a trial series of duplicate experiments is carried out, then the experimental results are statistically processed and the minimum required number of experiments is determined using formula (1):

$$n_{min} = \left(\frac{\sigma \cdot t}{\bar{R} \cdot k_t} \right)^2 \quad (1)$$

where \bar{R} – the average value, σ – the standard deviation of measurements; k_t – the required measurement accuracy in relative units, t – a variable value subject to the Student distribution.

For practical determination of the required number of concrete strength measurements, this method seems quite cumbersome. Therefore, the authors applied the approach of classical error theory [13]. According to the classical error theory, to calculate the number of experiments, it is necessary to know the values of the coefficient of variation v and the accuracy index P , which we replace with the sclerometer's certified instrument error Δ .

The probability of a random variable deviating from the mathematical expectation by no more than Δ according to [14-16]:

$$n_{min} = \left(\frac{\sigma \cdot t_{\alpha}}{\Delta} \right)^2 \tag{2}$$

where t_{α} – the value corresponding to the normalized Laplace function for the two-sided formulation of the problem $\bar{\Phi}(t_{\alpha}) = (1 - \alpha)/2$ at a given level of probability of error of the first type (reference value), Δ – the two-sided instrumental error in MPa (sclerometer passports). Taking the value $\alpha=0.05$, we obtain, according to (2), the following final expression:

$$n_{min} = \left(\frac{\sigma \cdot 1,96}{\Delta} \right)^2 \tag{3}$$

The analysis of the passports of sclerometers used in our country revealed a minimum error of ± 4.5 MPa, a maximum of ± 8 MPa. According to [17], the coefficient of variation is determined by the expression:

$$v = \frac{\sigma}{R}$$

from where, taking the coefficient of variation equal to the maximum permissible 13.5%, we obtain the expression for the standard deviation from the average strength of concrete:

$$\sigma = 0,135 \cdot \bar{R} \tag{4}$$

which are used in current calculations.

RESULTS

The obtained results are shown in table. 1

Table 1. Minimum number of experiments at $v=13.5\%$

Concr ete class	Nearest brand of concret e	Average compressive strength, \bar{R} , MPa	The mean square deviation by (4), σ	Minimum number of experiments (3)	
				$\Delta = \pm 4,5 MPa$	$\Delta = \pm 8MPa$
B3,5	M50	4,50	0,608	*	*
B5	-	6,42	0,867	*	*
B7,5	M100	9,63	1,300	*	*
B10	-	12,84	1,733	*	*
B12,5	M150	16,05	2,167	*	*
B15	M200	19,26	2,600	*	*
B20	M250	25,69	3,468	2	*
B22,5	M300	28,9	3,902	3	*
B25	-	32,11	4,335	4	*
B27,5	M350	35,32	4,768	4	*
B30	M400	38,35	5,177	5	2
B35	M450	44,95	6,068	7	2
B40	M500	51,37	6,935	9	3
B45	M600	57,8	7,803	12	4
B50	-	64,2	8,667	14	5

B55	M700	77,06	10,403	21	6
B60	M800	77,64	10,481	21	7

The “*” sign in the table means that instruments with such a measurement error should not be used to determine the strength of concrete of these classes (grades). Obviously, the minimum number of measurements depends on the strength of the concrete and the instrument error of the device used. A graphical representation of the obtained dependencies for various values of the coefficients of variation is shown in Fig. 1, 2. The maximum value of the coefficient of variation assumed in the calculations is 35% (0.35), which corresponds to the upper limit of values, after which the sample is considered heterogeneous [16], the minimum value is assumed to be 10% (0.1).

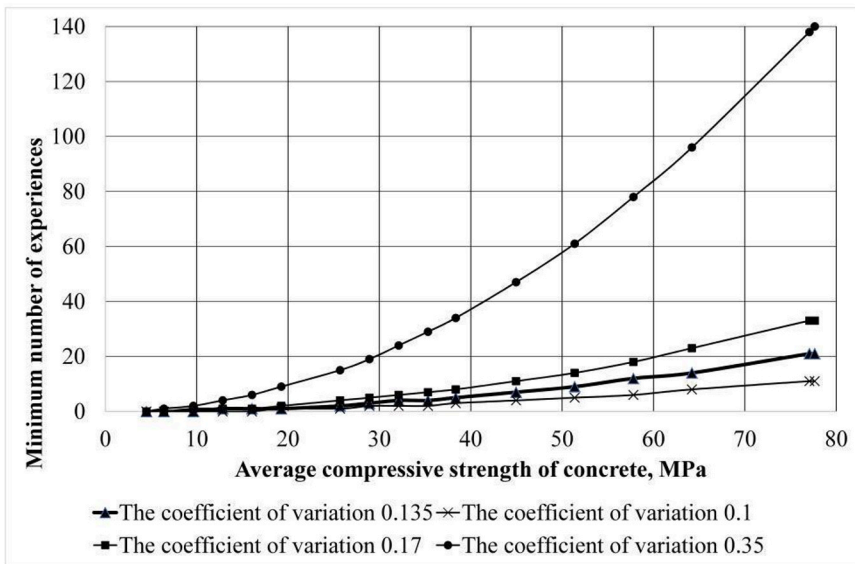


Fig.1. Dependence of the minimum number of experiments on the strength of concrete at various coefficient of variation for instruments with an error of ± 4.5 MPa

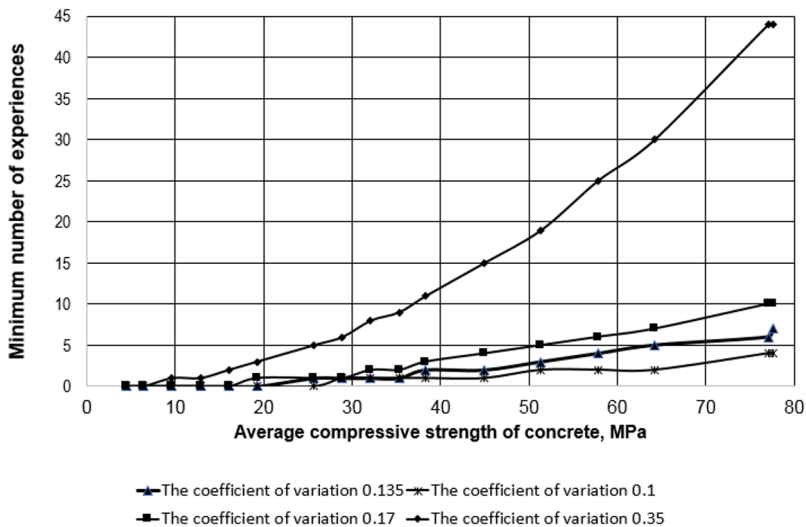


Fig.2. Dependence of the minimum number of experiments on the strength of concrete at various coefficient of variation for instruments with an error of ± 8 MPa

DISCUSSION

It is obvious that the minimum number of experiments is not a constant value and increases with increasing concrete strength at the same value of the coefficient of variation. As the coefficient of variation increases, the minimum required number of experiments also increases. For a normalized coefficient of variation of 13.5%, the calculated dependence of the minimum number of experiments on the average strength of concrete for the minimum and maximum instrument errors of the sclerometers used was obtained:

$$n_{min} = 0,0035 \cdot \bar{R}^2 - 0,0027 \cdot \bar{R} - 0,0312 \tag{5}$$

$$n_{min} = 0,0010 \cdot \bar{R}^2 + 0,0094 \cdot \bar{R} - 0,2248 \tag{6}$$

Expression (5) is valid for the minimum instrument error (coefficient of determination 0.99; expression (6) is valid for the maximum instrument error (coefficient of determination 0.98). The values obtained by (5) and (6) should be rounded to the nearest larger integer. Confidence intervals were also calculated, which allow estimating measurement outliers. For a normal distribution according to [14-16], the following is true:

- for the left edge of the interval $\bar{R} - (1,96 \cdot \sigma / \sqrt{n})$;
- for the right edge of the interval $\bar{R} + (1,96 \cdot \sigma / \sqrt{n})$.

Taking the minimum values of the number of experiments from Table 1, confidence intervals were determined for the normative value of the coefficient of variation. The data are summarized in Table 2.

Table 2. Confidence intervals for determining the strength of concrete ($v = 13.5\%$), depending on the instrument error of the Schmidt hammers used

Concrete class	Nearest brand of concrete	Average compressive strength, \bar{R} , MPa	Left edge of the interval, MPa		Right edge of the interval, MPa	
			Instrument error, Δ		Instrument error, Δ	
			$\pm 4,5$ MPa	± 8 MPa	$\pm 4,5$ MPa	± 8 MPa
B3,5	M50	4,50	*	*	*	*
B5	-	6,42	*	*	*	*
B7,5	M100	9,63	*	*	*	*
B10	-	12,84	*	*	*	*
B12,5	M150	16,05	*	*	*	*
B15	M200	19,26	*	*	*	*
B20	M250	25,69	20,88	*	30,49	*
B22,5	M300	28,9	24,48	*	33,32	*
B25	-	32,11	27,86	*	36,36	*
B27,5	M350	35,32	30,65	*	39,99	*
B30	M400	38,35	33,81	31,18	42,89	45,52
B35	M450	44,95	40,45	38,08	49,44	51,82
B40	M500	51,37	46,84	44,57	55,90	58,17
B45	M600	57,8	53,39	50,15	62,21	65,45

B50	-	64,2	59,66	56,60	68,74	71,80
B55	M700	77,06	72,61	69,35	81,51	84,77
B60	M800	77,64	73,16	70,79	82,12	84,49

CONCLUSION

1. The work shows that it is not entirely correct to accept a deterministic value for the minimum number of experiments for all grades (classes of concrete). This value depends on the average strength of concrete and the instrument error of the device used. For a given coefficient of variation, with increasing concrete strength, the required number of experiments increases.
2. For a coefficient of variation of 13.5%, calculated dependences of the minimum number of experiments on the average strength of concrete were obtained for the minimum and maximum instrument errors of the sclerometers used, determined by analyzing the passport data used in Russian organizations involved in the inspection of buildings and structures.
3. Concrete grades have been identified whose strength cannot be reliably determined by Schmidt hammers with the studied instrument errors.
4. The proposed method, which relates the strength of the material of the structure under study and the instrumental error of the device used, can be extended to other methods of determining strength by non-destructive methods and can be the basis of the technical specifications for the creation of devices for non-destructive testing of concrete.

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