A majorizing forecast of the earth population in a reduced resource-limited model

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Abstract. Three two-parameter phenomenological models of population growth dynamics of the Earth's population N leading to a conclusion about the existence of its limiting number N_{max} are briefly described. It is noted that the Ferhulst model, which assumes external resource constraints of exponential growth and leads to a logistic dependence N (t), has a maximum growth rate dN/dt at $N = N_{max}/2$ and is adequate to asexual method of reproduction. Two other models are associated with sexual reproduction, however, in Kapitsa model, it is believed that N growth restriction is due only to internal causes (the "demographic imperative" of Kapitsa). The authors develop a model in which resource constraints are reflected by the square of Verhulst multiplier. It is significant that in such a model (as in Kapitsa model) there is not only a maximum growth rate, but also a maximum relative growth rate (1/N) dN/dt at $N \approx N_{max}/2.7$ for Kapitsa model and $N = N_{max}/3$ in resource limited model. These features make it possible to estimate the value of N_{max} using statistical data on N(t) dependence. The analysis of transition from "microscopic" to macroscopic description indicates the possibility of transition to a reduced model in which for (1/N) dN/dt the square of Verhulst multiplier is preserved, and data processing allows using the value N_{max} = 14.7 billion. For this value N_{max} after calibration of the model parameters according to the beginning of 2018 a population forecast was made. Comparison with two forecasts based on UN data showed that calculations of the reduced model majorize them, exceeding, for example, in 2050 by 1.4%. The effect of explicit concern of the rate of population decline due to mortality, leading to decrease in the N_{max} estimates, is briefly discussed.

1 Introduction

The problem of the possibility of mankind existence on the Earth, under the conditions of its unlimited growth, was first specified by T. Malthus [1] and finally found its reflection in the model of exponential growth

$$N(t) = N(0)\exp(\lambda t)\,, \quad \lambda>0\,. \tag{1}$$

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Obviously, differential form of model (1) looks as

$$\frac{dN}{dt} = \lambda N. \qquad (2)$$

Certainly, T. Malthus, basing on the comparison of the data on population number growth (in geometric progression, equivalent to exponential growth (1)) and the data on food production growth (in arithmetic progression) came to conclusion about the inevitability of crisis phenomena and the necessity of limitation of population number in order to prevent social commotions. These qualitative conclusions gave base for models of «demographic cycles» (see, for instance, [2-4]) and of accompanying economical and political processes.

Let us note that model (2) leads to infinite value N in the case of passage to the limit $t\to\infty$. As the infinite value N clearly indicates incompleteness of the model, the question of its improvement arises. A simple analytic variant of model modernizing (2) was proposed by P. Verhulst [5] who rightfully supposed that growth had to stop after attainment of some ultimate value K, conditioned by shortness in resources. According to [5], it is realized by insertion of additional factor into the right part of (2):

$$\frac{dN}{dt} = \lambda N(1 - \frac{N}{K}). \qquad (3)$$

It is obvious from (3) that at N = K the rate of change dN/dt converts into zero, and if the initial value of N(0) < K, then $K = N_{max}$ is maximum value at which N asymptotes ($t\to\infty$). The expression (3) corresponds to the so called logistic model (logistic curve) that was repeatedly discussed and used for various processes descriptions (see, for instance, [6-8]). Note that, according to (3), the maximum value dN/dt is reached at $N = \overline{N} = K/2$, and the relative rate of change N^{-1} dN/dt has no extremes.

The law (2) is naturally well coordinated with ways of asexual reproduction known in biology and leading to population number growth [8]. According to (2), the rate of increase is proportional to actual number of individuals *N*. Such proportionality is quite rational, if one considers that each individual can reproduce (i.e., by budding) irrespectively of the others. Correspondingly, the value K is called the capacity of ecological area. It's clear that the law (2) and its integral analogue (1) are not plainly immanent to human society as the growth of its number is based on sexual way of reproduction.

Processing of statistical data on dynamics of population lead H. Forster [9] to a conclusion on hyperbolic integral dependence

$$N(t) = \frac{C}{t_1 - t}, \quad (C = const > 0, \quad t_1 = const > 0), \qquad (4)$$

resulting in singularity N(t), reached (at $t\to t_1$) for finite amount of time t_1 (t_1 corresponds to the year 2026 of actual calendar). It is easy to match integral dependence (4) and differential law

$$\frac{dN}{dt} = \delta N^2, \qquad \delta = \frac{1}{C} > 0. \qquad (5)$$

It should be emphasized that perception of empiric law (5) as a base for macroscopic phenomenological description in terms of model substantially different from (1) lead S.P. Kapitsa [10] to radical postulates.

«1. All population of the Earth id considered to be integrated, strongly unified developing system.

2. Growth rate is proportional to the square of the world population. It is conditioned by universal interaction that is independent of external resources being an inner process.

3. The interaction is based on multiplying and distribution of information of any kind.

4. A coherent and self-sufficient population is a group consisting of ~ 60000 people».

In contrast to the concept of P. Verhulst connected with resource constraints, S. Kapitsa, in order to avoid singularity, first presents the right part of law (5) as singular function of time using integral solution (4)

$$\frac{dN}{dt} = \delta \frac{\mathrm{C}^2}{(\mathrm{t_1} - \mathrm{t})^2}\ , \tag{6}$$

and then an addend τ^2 is artificially introduced into denominator (6),

$$\frac{dN}{dt} = \delta \frac{\mathrm{C}^2}{(\mathrm{t_1} - \mathrm{t})^2 + \tau^2} = \frac{1}{\delta[(t-t_1)^2+\tau^2]}, \qquad t_1 = t_0 + \frac{1}{\delta N(t_0)}, \tag{7}$$

where time τ (measured in years) is interpreted as characteristic inner scale, commensurable with life duration of one generation. At the same time it is emphasized that the transition to maximum population number is governed not by external resource shortness, but is immanent to the dynamic system itself (Kapitsa's «demographic imperative»).

Integrating of the expression (7) results in a smooth function

$$N(\mathrm{t}) = \frac{\mathrm{C}}{\tau} arc\cot\left[\frac{\mathrm{t_1} - \mathrm{t}}{\tau}\right] = \frac{\mathrm{C}}{\tau}\{\pi - arc\cot\left[\frac{\mathrm{t} - \mathrm{t_1}}{\tau}\right]\}\ . \tag{8}$$

Let us recall that function $f(x) = arc\cot x$ takes its maximum value π (at $x \to -\infty$) and the minimum one – 0 (at $x \to \infty$). At $x = 0$ we get $f(0) = \frac{\pi}{2}$ and maximum of the function change rate. Respectively, the function $(\pi - f(x))$ takes its maximum value π at $x \to \infty$ and the minimum one 0 at $x \to -\infty$. The function $(\pi - f(x))$ beside the maximum of growth rate in the point $x = 0$ possesses at $x \approx -0.43$ also a maximum of relative growth rate that is described by expression:

$$\frac{1}{(\pi - f(x))}\frac{d(\pi - f(x))}{dx} = -\frac{1}{(\pi - f(x))}\frac{d\,f(x)}{dx}\,.$$

Let us note that $(-x) = \pi - f(x)$, and the condition of extreme (first-order derivative from relative rate is equal to zero) is reduced to equality

$$\mathrm{x}f(x) = -1/2.$$

In the point of maximum for relative growth rate $f(x) \approx 1.164698268$. Consequently, $f(x)/\pi \approx 0.370735$. Then, using value $\widetilde{N}$ in the maximum point of relative growth rate, the extreme value is set up by a ratio

$$N_{max} \approx \frac{\widetilde{N}}{0.370735} \approx 2.69734466\widetilde{N} \approx 2.7\widetilde{N}. \tag{9}$$

In fact, acute deviation from the law of hyperbolic population growth, connected with birth-rate falling, got the name of demographic transition.

Plainly phenomenological treatment of the ratio (5) as a macroscopic trend, without any attempt of its «microscopic» reasoning taking into account the human factor, had generated, to our opinion, the former cited postulates of S.P. Kapitsa. The atmosphere of sensation after the work [9] made its contribution into the popularity of the model of hyperbolic growth (see, for instance, [11]). And the macro-description wad assumed noncritically, without analysis of its «microscopic» grounds.

2 Two-parameter resource-limited model adequate to the sexual way of reproduction

It is easy to show, however, that it's quite natural to come to macroscopic description (5) basing on biological perceptions of the sexual way of reproduction for which using (5) seems logical. Indeed, the population growth is the result of interaction of two collectives of male and female sex (approximately equal in number), and so the growth rate should be proportional to the product of their numbers. For instance, in biology, for localized population areas, this variant is confirmed and linked with the rate of partners' contacts [8].

In phenomenological macro-description of the Earth population growth rate the law (5) may be used with expectations that, owing to biological fundamental principle, it will better reflect the nature of process than the linear law (1) does. However, transfer of the conclusions relevant for local areas to the planetary level requires special study. Thus, the ideas of S.P. Kapitsa about mankind as a total strongly bound by informational interaction look not enough decisive, at least in the context of the processes of population number change.

As an alternative we shall further discuss, following [12-14, 17-19], a diametrically opposite statement: the leading role in forming of the pairs of potential sexual partners belongs to rather narrow communication spheres (CS). Indeed, effective sexual partnership is accomplished, in overwhelming majority, in geographically localized regions, substantially less than continents and states. But even within cities (large, medium and small) the partner choice, as a rule, takes place among a limited group of people, with whom confident relations are possible, owing to steady communication in social sphere (studies, work, membership clubs). Broadening of contacts in the Internet, owing to limitation of free time, does not principally (by the order of magnitude) change the CS.

Fundamental characteristic for the change of population number is effective (averaged) contribution of one pair into population reproduction. This characteristic is the first one to reflect the influence of all social and economical factors on making the decision to have children. It is the characteristic reflecting the human factor in concentrated and, more importantly, in quantitative form. Thus, it is necessary to inquire the transition from «microscopic level» of one pair of sexual partners through «mesoscopic level» of CS to macro-description (5). Solution of this problem will help to discover the structure of phenomenological constant δ in (5), i.e. to understand the way it is connected with characteristics of «micro»- and «meso»-levels.

Let us show, basing on these ideas, how to get an estimate of the relative rate of population growth $\frac{1}{N}\frac{dN}{dt}$, corresponding to the observed value.

Let N_{mi} and N_{wi} be quantities of persons of male and female sex in any CS with number i (i=1, 2, 3…., n), then

$$N_i = N_{mi} + N_{wi}, \qquad N_{mi} \approx N_{wi} \approx \frac{N_i}{2}, \tag{10}$$

$$\frac{dN_{wi}}{dt} = \delta_{wmi} N_{wi} N_{mi}, \quad \frac{dN_{mi}}{dt} = \delta_{mw,i} N_{wi} N_{mi}, \quad \delta_{mw,i} = \delta_{wm,i} = \delta_{0i}, \tag{11}$$

$$\frac{dN_i}{dt} = \frac{dN_{mi}}{dt} + \frac{dN_{wi}}{dt} = +2\,\delta_{0i} N_{mi} N_{wi} \approx \frac{1}{2}\delta_{0i} N_i^2. \tag{12}$$

Then for CS of *i-th* number writing (5) takes the form of:

$$\frac{dN_i}{dt} = \delta_i {N_i}^2. \quad (13)$$

The comparison of (12) and (13) gives

$$\delta_i \approx \frac{1}{2}\delta_{0i}. \quad (14)$$

Let us show now, using the form of writing (5), how to accomplish the transition from «microscopic» description (in terms of one pair of partners) to macroscopic description for the Earth population through intermediate level of CS. Passing from CS level to the total Earth population, we shall consider for simplicity, that the numbers N_i for all CS are the same and make N_S, and so the values $\delta_i = \delta_S = 0.5\delta_0$. Then the total number of CS groups is

$$n_S = N/\ N_S. \quad (15)$$

Summarizing of left and right parts of equations (13) from 1 to n_S leads to a ratio

$$\frac{dN}{dt} = \delta_S\, n_S\, {N_S}^2\ . \quad (16)$$

It is important to point that the transition from the form of equation (16) to the form of writing (5) means explicit inclusion of value n_S into the structure of parameter δ, namely:

$$\frac{dN}{dt} = \delta N^2, \qquad \delta = \frac{\delta_S}{n_S}. \quad (17)$$

As the value n_S is expectedly large number, parameter $\delta \ll \delta_S,$ and such spasmodic reduction is an obvious pay for the formal introduction of the square of total Earth population to the right part of (17). This condition clarifies the cause of illusory view of S.P. Kapitsa on the role of Earth population as a total, in the law (17).

Let us state now the connection of value δ_S with the number N_S and fundamental parameter δ_0. Let symbol P_{ef} be the effective number of heterosexual (mw) pairs P_{ef}, that is the component of the total number of (mw) pairs ranged by age (including old men and infants) $P \approx N_S/2$, the partners of which intend to have children. We'll mark the share of these pairs as $p = P_{ef}/P$.

For qualitative (order of magnitude) estimates we'll further tentatively consider that share of P_{ef} in the total number of pairs P makes 20%, and therefore, $p = 0.2,\ P_{ef} = 0.1N_S$.

Supposing that n_S changes relatively slow, we'll associate N, in general, with growth of N_S. The rate of change of N_S value, from one side, according to (13), is connected with the form of writing (5)

$$\frac{dN_S}{dt} = \delta_S {N_S}^2, \quad (18)$$

and from the other side, is connected with the number of effective pairs within SC.

In the extreme case of one pair using the form of writing (5) means that

$$\frac{dN_1}{dt} = \frac{1}{2}\delta_0 {N_1}^2, \quad (19)$$

where $N_1 = 2$, or, taking into account (14),

$$\frac{dN_1}{dt} = \frac{1}{2}\delta_0 4 = 2\delta_0. \tag{20}$$

Let us estimate the value δ_0, supposing that a pair of heterosexual partners during reproductive period Δt_r (for instance, Δt_r = 30 years) of their life t_ℓ gives birth on the average to m children (for instance, m = 3 for extended reproduction), then, in the extreme case of one pair, we get

$$\Delta N = 3 = 2\delta_0 \Delta t_r, \tag{21}$$

$$\delta_0 = \frac{m}{2\Delta t_r} = \frac{3}{60} = 0.05(1/\text{year}). \tag{22}$$

That means that

$$\frac{dN_S}{dt} = P_{\text{ef}}\frac{dN_1}{dt} = 2\delta_0\, P_{\text{ef}} \approx \delta_0\, p\, N_S\,, \qquad \frac{1}{N_S}\frac{dN_S}{dt} = \delta_0\, p. \tag{23}$$

From (23) it is obvious that, at $p = 0.2$, the relative growth rate

$$\frac{1}{N_S}\frac{dN_S}{dt} \approx 0.2\delta_0 \approx 10^{-2}, \tag{24}$$

i.e. makes 1%.

The relative growth rates are observed just near 1%. It should be so, because we consider $N = n_S N_S$, and at fixed (or slowly changing) numbers n_S

$$\frac{dN}{dt} \approx n_S\frac{dN_S}{dt}, \quad \frac{1}{N}\frac{dN}{dt} \approx \frac{1}{N_S}\frac{dN_S}{dt}\,. \tag{25}$$

It follows that our estimations comply with the real state of affairs. Let us note that using the form of writing (18), that at the level of CS corresponds to writing (5), from comparison of (18) with (23) we get

$$\delta_S = \frac{\delta_0\, p}{N_S}. \tag{26}$$

Then from (17) we obtain a ratio

$$\delta = \frac{\delta_S}{n_S} = \frac{\delta_0\, p}{N_S n_S}, \tag{27}$$

that uncovers approximately multiplicative structure of the parameter δ.

In such a way, the model, in approximation of non-overlapping communication spheres with equal quantities and time dependencies $N_S(t)$, allowed the transition from the contribution of one pair of partners into the growth of population number to the variant (5) with dependency of growth rate on the square of total population number $N = N_s n_s$, where n_s is the quantity (a large number) of communication spheres. There are grounds to suppose [14], that CS are relatively compact (with numbers $N_S \sim$100-1000 persons).

Thus $N = n_S N_S$, then, according to (27), $\delta \sim 1/N$ and model (5) may be reduced to model (2). Therefore, S.P. Kapitsa's postulates are too non-critical.

Keeping the form of model (5) as one reflecting the mechanism of sexual reproduction, allows [14] to use a variant including the square of P.Verhulst multiplier as a resource-limited model

$$\frac{dN}{dt} = \delta N^2(1 - \; N/K)^2. \tag{28}$$

Let us emphasize that for models (3), (8), (28) a maximum of growth rate dN/dt at $N = \overline{N} = N_{max}/2$ is characteristic. In fact, for models (8) and (28) relative growth rate N^{-1} dN/dt also has its maximum at $N = \widetilde{N} < \overline{N}$: $N_{max} \approx 2.7\widetilde{N}$ for model (8) and $N_{max} = 3\widetilde{N}$ for model (28).

The presence of marked maximums gives the opportunity to estimate the extreme value of the Earth population basing on the data on N growth dynamics.

3 The data on dynamics of the Earth population growth and the forecast of its number in a reduced model

According to the data[1] of UN Department of economical and social problems (the population section), time dependence of the relative growth rate N^{-1} dN/dt has two peaks (see Fig.1): in the middle of 1969 ($N \approx 3.52$ billion) and in 1986 (at $N \approx 4.9$ billion). (CountryMeters. [Electronic resource] URL: http://countrymeters.info/ru/World#historical_population (Date of inquiry 16.07.2019).)

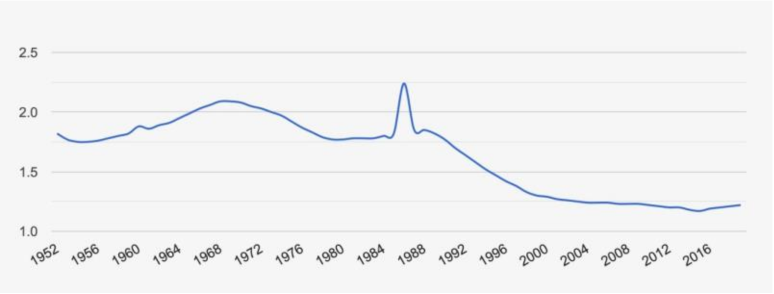

Fig.1 The data on the relative growth rate of the Earth population

Approbation of models (8) and (28) taking as $\widetilde{N} = 3.52$ billion was held by [13] for time interval from 1968 till 2018, and some preference was given to the model (28) with value $N_{max} = 10.56$ billion. The second possible value $N_{max} = 14.7$ billion (taking $\widetilde{N} = 4.9$ billion) was considered less probable.

In [14] it was marked that the dependence N(t) was described upon half-century interval quite satisfactory (accuracy of the order of 1%), but the values of relative growth rate had noticeable difference. An assumption was put forward that, at the reached values of N it was feasible to go over from model (28) to a reduced variant (by change $\delta \rightarrow \lambda/N$):

$$\frac{dN}{dt} = \lambda N(1 - N/K)^2, \tag{29}$$

which keeps, nevertheless, the memory about the sexual way of reproduction (owing to the square of P. Verhulst multiplier). A maximum d*N*/dt at $\overline{N} = N_{max}/3$ is characteristic for model (29). Additional analysis of the data [1] showed that in 1986 there was a maximum of growth rate as well. Therefore, there are grounds for choice of $N_{max} = K = 3 \cdot 4.9 = 14.7$ billion in the reduced model (29) also. Calibration of the parameters of the mentioned models accounting resource constraints was made in [14], basing on proposed values of relative growth rate 1.2% and population number 7.57695 billion on January 1, 2018, and the forecasts of *N* were made, including the long-term ones, referring to the beginning of the year (CountryMeters. [Electronic resource] URL: http://countrymeters.info/ru/World#historical_population (Date of inquiry 16.07.2019).).

Owing to the fact that in 2018 the long-term forecasts appeared at the site [1], though without naming the extreme value N_{max}, it is interesting to compare our calculated data with the calculation of models made by method of [15]. In [15] the level of birth-rate, mortality rate and migration flows are predicted for every state. The integral forecast is given as well, being of interest for comparison with our results, because it does not depend on migration. Here we restrict ourselves by a forecast in terms of model (29), doing calibration by data [1] on the beginning of 2018 (at K = 14.7 billion):

$$\frac{dN}{Ndt} = 0.0121\left(\frac{1}{\text{year}}\right), \quad N = 7576951385, \quad \lambda \approx 0.05153333185\,(\frac{1}{\text{year}})\,. \quad (30)$$

Then, on January 1, 2019 the calculation by model (7) gives N ≈ 7.6680056 billion, that is close to estimation [1]: 7.6691091 billion. The further forecast in [1] is made with interval of 5 years on the middle of the year (July, 1) until 2100, therefore, for the ease of comparison, in Table 1 our data are also given on July, 1 of the corresponding year (till 2005 – annually, then – each 5 years). Besides that, in two last columns the forecast [2], accomplished by Worldometers, somewhat different from data [1] is given. This forecast is based, beside the data of UN Department of Economical and Social Problems (Population Section), also on the data of the Center of International Programs of US Sensus Bureau (Population Section). It's worth mentioning that data [2] have some difference from [1] also at the interval (1951 – 2018), as it's evident from Fig. 2.

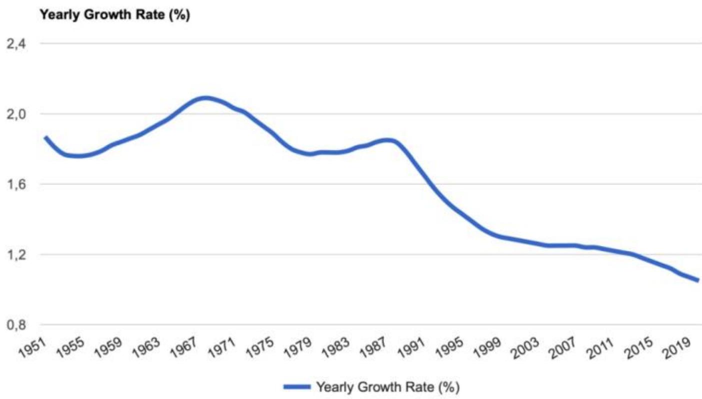

Fig. 2. The data on the Earth population relative growth rate

Table 1. Population number forecasts

Forecast of model (29) at K = 14.7 billion			Data [1]		Data [2]	
Dates	$\frac{1}{N}\frac{dN}{dt}(\%)$	N (billion)	N (billion)	$\frac{1}{N}\frac{dN}{dt}(\%)$	$\frac{1}{N}\frac{dN}{dt}(\%)$	N (billion)
2019	1.16	7.713		1.22	1.07	7.715
2020	1.13	7.802	7. 797	1.67	1.05	7.795
2021	1.11	7.890				
2022	1.08	7.977				

2023	1.05	8.062				
2024	1.02	8.115				
2025	1.00	8.229	8. 187	5.00	0.98	8.186
2030	0.88	8.624	8. 553	4.47	0.88	8.551
2035	0.78	8.989	8. 895	3.99	0.79	8.893
2040	0.69	9.324	9. 212	3.57	0.70	9.210
2045	0.61	9.632	9. 506	3.19	0.63	9.504
2050	0.55	9.915	9. 774	2.81	0.56	9.772
2055	0.49	10.174	10. 013	2.45		
2060	0.44	10.412	10. 224	2.11		
2065	0.39	10.631	10. 411	1.83		
2070	0.36	10.833	10. 577	1.59		
2075	0.32	11.018	10. 723	1.38		
2080	0.29	11.189	10. 850	1.18		
2085	0.27	11.347	10. 959	1		
2090	0.25	11.494	11. 051	0.84		
2095	0.22	11.630	11. 127	0.69		
2100	0.21	11.756	11. 185	0.52		

4 Discussion of the results

As follows from Table 1, the most substantial peculiarity of the forecast [1] is the increase of relative growth rate (till 5% in 2025) in the nearest years with following monotonous decline. The base of this record (for the period after 1951) increase of $N^{-1}\,dN/dt$ in 2025, according to [1], should be made by demographic raise in India (4.96 %), and also by population growth in a number of large countries listed in the order of decreasing of their population number on the beginning of 2019: Indonesia (4.6%, 268 mln), Pakistan (8.83%, 203 mln), Nigeria (13.36%, 200 mln), Bangladesh (5%, 168 mln), Mexico (5.43%, 133 mln), Ethiopia (11.85%, 109 mln), Philippines (7.26%, 102 mln), Egypt (8.29%, 99 mln), Congo (16.44%, 86 mln). There is a number of countries with high rate of growth, but with population substantially less than 100 mln (for example, SAR (5.23%, 53 mln), Sudan (12.54%, 43 mln), Niger (20.7% , 22 mln). From the point of view of estimation of extreme population number N_{max} in terms of simple two-parameter models, the appearance of such maximum could be indication of acute increase of N_{max}.

However this forecast is not confirmed by the data [2], both in total (0.98 % in 2025) and for individual states, for example, for India (0.96%), Pakistan (1.71%), Nigeria (2.54%). As a result, in [2] the continuation of monotonous decline of relative growth rate is predicted, moreover, as seen from Table 1, values N^{-1} dN/dt appear close to the data of model (29), especially beginning with 2025. At the background of stated disagreement of the forecasts [1] and [2] substantial deviations of the dependencies of population number should be expected. However, by the data of Table 1, this difference is not large, that is reflected in practical agreement of the data of reaching the borderline values shown in Table 2.

The comparison of the forecast values of model (29) with the data of calculations [1, 2] demonstrates growing with time majorizing of the estimates of model (29). For instance, already in 2025 there is majorizing by 42 mln, that corresponds to 0.5%, and by 2035 – already 94 mln (1%), by 2045 – 125 mln (1.3%), by 2055 – 162 mln (1.6%), by 2075 – 295 mln (2.7%), by 2085 – 388 mln (3.4%), by 2100 – 570 mln (4.85%). This majorizing is reflected in surpassing attainment of borderline values of population number (Table 2). Thus, estimates of the model (29) play the role of majorizing relatively to [1, 2].

Table 2. Forecast dating of the achievement of boundary values of the Earth population

N(billion)	8	9	10	11	12	13	14
Data by model (29)	2022	2035	2052	2074	2111	2184	2442
Data by forecasts [1] and [2]	2023	2037	2055	2088			

Note: in Table 2 the years are named during which the boundary values are attained (without detailing – months and days).

The used two-parameter model is extremely simple and aimed only at the estimation of the Earth population number, without explicit account of birth-rate and mortality rate. However for adequacy of the estimations reliable enough statistic information is needed. Only a year ago the authors perceived the data [1] as the most reliable, nevertheless, the existence of forecast [2] in the informational field demonstrates the ambiguity of processing of the data on the Earth population number. In this connection two questions come forward: not only about choice of the most accurate two-parametrical models, but also about the reliability of statistic data concerning real population dynamics. In particular, verification of short-term forecast on the maximum of relative growing rate of population number in 2025 is interesting.

It goes without saying that long-term forecasts without possibility to foresee the variants of technological revolutions (becoming more frequent) and their social consequences bear conditional character. High rate of forecast uncertainty marked in [15] is not occasional. Nevertheless, simple models are useful, both for processing the observed dynamics of population number and for comparative analysis of the data obtained in more complicated models.

The fact that proposed reduced model (29) majorizes (with relatively small exceeding) the data of forecasts based on method [15], is substantial for accessible to everyone, extremely easy and independent verification of the proposed forecasts of population number. Forecast probability is the most important factor for planning of the necessary resources of development and anticipating reaction on the problems generated by overpopulation of the planet. A summary [16, 17] (in terms of the ideology of resource constraints declared by the Roman club in 60-th years of XX century) is well-known; it testifies about alarming tendencies of environmental changes and modern biosphere.

As to methodological problems, it should be noted that there are no serious grounds for declaring a specific «demographic imperative» applied to the model of S. P. Kapitsa. The dependency (8) itself, of course, may be used as one of phenomenological models reflecting the sexual way of reproduction of mankind. In this connection the interpretation of specific value of the fitting parameter τ bears secondary character. The well-timed account of the change of fundamental parameter δ_0 (as well as parameter p in (27)), reflecting in concentrated form the influence of all factors of material world on the decision to have children is much more important. It is well known, for example, that the increase of population education results in decline of growth rate. The models aimed on the relation between population and the peculiarities of economic development (seem for instance [18, 19]) also deserve attention as well as discussion on the opportunities of synergetic description [20].

It is important to note that the discussed model estimates supposed the variant of stabilization of the Earth population along with tendency to asymptotic stationary value N_{max}. However such variant cannot be considered neither optimistic nor the only feasible one. Really, stabilizing of the population at a level of 9.5 billion, or 13.23 billion according to model (8) as well as at a level of 10.56 billion, or 14.7 billion according to models (28) and (29) most likely exceeds the values N_{max} compatible with steady functioning of the Earth biosphere. **Therefore** such stabilization does not solve the problem of planet over-population (at least within the limits of existing paradigms and technologies). However one cannot consider favorable the sceneries of population collapse sue to supposed fast birth-rate falling [17]. Especially, if we take into account the arising analogies between the tendencies of development of modern society and the results of experiments [21-25] on modeling the «mouse paradise» evolution.

This serious item deserves separate discussion. Nevertheless, we'll mark that ignoring the explicit account of mortality factor in models (8), (28), (29) supposes, from one side, its small role at the background of intensive population growth, and from the other side, at setting parameter δ_0 one means really survived progeny (implicitly accounting for children's mortality). However for more realistic description, especially for reduced reproduction, one should explicitly account for the rate of population diminution, adding a summand – dN, where d > 0, to the models.

For example, modifying the model (29), we get

$$\frac{dN}{dt} = \lambda N(1 - \ N/K)^2 - dN. \tag{31}$$

It follows from (31) that at fixed values of parameters λ and d, not unreasonable stationary (at $\frac{dN}{dt} = 0$) solution $N_{stat} > 0$ exists if $d < \lambda$:

$$N_{stat} = \widetilde{K} = K(1 - \sqrt{\frac{d}{\lambda}}) \ . \tag{32}$$

From (32) it is obvious that $\widetilde{K} < K$, thus majorizing of population forecasts by our estimates is quite expected. It's also clear that, at keeping relatively low values of d (for instance, because of high level of available medical care of population), but at social directions leading to λ decline, the decrease of $\widetilde{K}$ until $\widetilde{K} \rightarrow 0$ at $\lambda \rightarrow d$ is inevitable. Certainly, understanding of the situation is able to stabilize the process of depopulation at some level $\widetilde{K}$ corresponding to high level of life quality, still compatible with stable situation in the Earth biosphere.

5 Conclusion

The analysis fulfilled had shown that using the reduced variant of model (28) in form (29) at the value of extreme population number 14.7 billion leads to estimates majorizing the forecasts of population number based on UN data that are in free access.

The results of our calculations of relative rate of population number growth (monotonously declining with time) appear close to the forecast of Worldometers [2] taking the information of US Sensus Bureau as additional source. At the same time the data of

Countrymeters [1] indicate an increase of growth rate expected in the nearest years (with decline after 2025). Realization of such forecast may require adjustment of the model and change of parameter calibration. However until reliable statistic data confirm (or disprove) the short-term forecast [1], or some new factors substantially affecting system dynamics appear, the data of model (29) may be used as acceptable for comparison of landmarks (with possible corrections (31)).

It is clear that, having reliable regional statistics, such models may be used for regional forecasting with obligatory addition, concerning migration contributions into the total number of regional population.

Acknowledgements

The authors are thankful to S. A. Nephyodov and to untimely deceased A.I. Kuzmin for discussions of the work and useful recommendations.

P.S. The writing of the article was initiated by Professor A.I. Kuzmin, whose untimely departure violated plans for joint research. The article was completed by us in 2019. Despite this, the described methodology, as well as the predictions made, remain, according to the authors, their relevance.

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