The creation of quasigeoid model by local method in the territory of the Republic of Armenia

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Abstract. The structure of the State altitude network of the Republic of Armenia, the adjusted observational data, as well as the obtained results of the accuracy assessment have been studied. Taking into account the available geodetic data, equipments, financial capabilities and professional skills of specialists in the country, it was accepted to create a quasigeoid model by local method. In 2008 a quasigeoid model of the territory of the Republic of Armenia has been created with the accepted method. 2015-2017 in order to update the model, the altitude points of quasigeoid were condensed. As a result, an updated model of quasigeoid was obtained. The value of estimated accuracy is up to 6 cm. Taking into account the current geopolitical situation of the Republic of Armenia, as well as the requirements of the instruction, it is proposed to make changes to the structure of the State altitude network of the RA and reobserve the I class levelling network according to the new project. This will provide basic geodetic altitude data, necessary for the development of the country's economy.

1 Introduction

The State geodetic altitude network of Republic of Armenia (RoA) was an integral part of the USSR State Geodetic altitude network. In the Caucasus region the main purpose of creating the first geodynamic polygon was to study the leveling changes between Black and Caspian seas, also to have observations of the earth vertical movements caused by man-made phenomena on the Apsheron peninsula, as well as to predict earthquake based on earth crust movements of the region In 1910 the first leveling line was created by the Military Topographic Corps of USSR. The levelling lines performed through the cities: Tbilisi - Kirovakan (Vanadzor) - Spitak - Leninakan (Gyumri) and crossed the epicenter of the Spitak earthquake. The second leveling line was passed through Aghstafa-Sevan-Mkhchyan. The third leveling line was passed Akhaltsikhe - Akhalkalaki - Leninakan (Gyumri) - Mkhchyan. In 1938-1940 the dual measurements were carried on benchmarks of the same levelling lines. Additional ground and rock benchmarks installed in those

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directions. During 1953-1974 period dual leveling measurements were carried out for times on the first, second and third levelling lines.

After the declaration of independence of the Republic of Armenia (in 1991 September) the problem is set, to update and develop the field of geodesy and cartography according with the requirements of normative legal and technical documents of the RoA. For that purpose, in 2001 "Geodesy and Cartography Center" CJSC carried out the study of the existing state geodetic and altitude networks in the territory of the Republic of Armenia. After analyzing the study results it was found out that 78% of the plane network points were preserved, which are mainly located on the heights and 55.5% benchmarks of altitude networks. Based on the results of the research, it becomes necessary to reconstruct the networks of State geodetic plan and the State geodetic altitude using modern satellite positions and electronic technologies.

Materials and Methods

Analyzing the study results of State geodesic altitude network (SGAN), it was found out that in territory of the RoA the first class levelling network is consists of 5 open polylines, which continued through the territories of Georgia and Azerbaijan. The 5 polylines are:

1. Masis - Armavir - Karakert - Maralik - Gyumri - Bavra, continuing to Georgia. In that line 38 benchmarks have been preserved,

2. Masis - Yerevan - Hrazdan - Sevan - Dilijan - Ijan - Krivoy - Most - Aghstev Reservoir, continuing to Kazakh. In that line 48 benchmarks have been preserved,

3. Masis - Artashat - Ararat - Yeraskh, from where one line continued to Nakhichevan, and the other to Yeğhegnadzor - Vayk - Shaki, after which it continued to Goris - Kapan - Kajaran – Meghri with II class leveling. In that line 60 benchmarks have been preserved,

4. Gyumri - Spitak - Vanadzor - Dzoraget - Alaverdi - Ayrum, continuing to Georgia. In that line 32 benchmarks have been preserved,

5. Sevan - Chambarak - Vardenis - Jermuk - Vayk, where 16 benchmarks have been preserved.

It is important to mention, that five lines of the I class levelling had been calculated and adjusted within I class leveling polygons of the former USSR and now it is impossible to do the same, so I class levelling lines were planned as a closed polygons, which allows to do calculation and adjustment altitude network of RoA, as an independent network [1-13].

For developing the altitude network scheme, the following premises had been taking into account:

• the scheme was made based on results of preserved benchmarks getting from the study and research information of altitude network in 2001,

• the railway and road networks of RoA had been studied on topographic maps at 1:200 000 scale, than I class main levelling lines were performed parallel with that networks. The II class levelling lines have been served as a density of I class levelling network,

• in order to determine normal heights with gravity corrections, the gravity measurements should be done on the benchmarks and marks of I and II class levelling.

The scheme polygons of I class leveling altitude network has been developed as forms of 8 polygons in RoA according to above-mentioned premises and principles (Fig.1).
The 42 base points of National geodetic network (NGN) of 0-class and 1, 2 classes have been included in the I and II levelling classes of State altitude network. Through these points, the connection is created between 1977 Baltic and World Altitude Systems; it means to have connection between normal and geodetic heights. The fundamental benchmark № 417 was used as an initial height (located in the Gyumry area) during the adjustment of reconstructed altitude network in the territory of the RoA, and as a control point, mark № 417 was used (located in the Vanadzor area). The selection of benchmark № 417 and mark № 418 is conditioned by fact that they have not been altered in heights over time; they preserved both the exterior form and interior structures. The total length of I class levelling 1584.72 km, II class levelling 940.57 km and connected lines of 0-class base points of NGN have 101.79 km length. The I class levelling was carried out in forward and reverse directions by two lines with changing levelers horizon. The I and II class State altitude networks (SAN) have been adjusted separately, then adjustment has done combined. The I class levelling network encompassed: 15 nodal points and 23 lines, as a result 8 closed polygons and 1 open polyline. The adjustment have been done by Local X. Positionin Sistem+ software. As a result, for each section there are obtained misclosure and permissible value. In polygones, the permissible closing error calculated by following formula:

\[ F_{\text{perm}} = \pm 3 \text{mm} \sqrt{L} \quad , \]

The elevations of I class levelling, which obtained from I and II lines (forward and reverse direction) compared. Between them (d1, d2, d3, d4, d5) permissible misclosure have been calculated:

- \( \pm 2 \text{mm} \sqrt{L} \) - if the average number of the station within 1 km is less than 15,
- \( \pm 3 \text{mm} \sqrt{L} \) - if the average number of stations within 1 km is greater than 15,
where \( L \) is the length of section in km. On the stations, the permissible misclosure of average evaluations \((d_6)\) forward and reverse lines calculated by formulas \(\pm 3\text{mm} \sqrt{L} \) and \(\pm 4\text{mm} \sqrt{L} \) respectively. For 1 km line mean-square random errors have been calculated by the following formulas:

\[
\eta_1^2 = \frac{1}{4n} \left[ \frac{d_5^2}{r} \right], \quad \eta_2^2 = \frac{1}{4n} \left[ \frac{d_6^2}{r} \right],
\]

(2)

Where, \( n \) - the number of sections of the line; \( d_5 \) - difference between average elevations of the I and II class levelling lines; \( d_6 \) - the difference between average elevations of the forward and reverse lines and \( r \) is the section’s length.

For 1 km line mean-square systematic errors calculated by the following formula:

\[
\sigma^2 = \frac{1}{4[L]} \left[ \frac{s^2}{L} \right],
\]

(3)

where \( L \) is the length of line, \( S \) values taken as the difference of a straight line ordinates systematically plotted on a corresponding section of the accumulation differences curve of uniform accumulation.

The II class levelling carried out forward and reverse lines. The II class levelling network had been created inside I class levelling network, system is consisted of 23 close polygon and 28 polylines. Benchmarks and marks of I class levelling used as an initial network had been created inside I class levelling network, system is consisted of 23 close polygon and 28 polylines. Benchmarks and marks of I class levelling used as an initial points for adjusting the II class levelling network.

As a result of the adjustment we have misclosure and permissible value for each section. Permissible misclosure has been calculated by following formula:

\[
F_{\text{perm}} = \pm 5\text{mm} \sqrt{L},
\]

(4)

The elevations of II class levelling, which obtained from I and II lines (forward and reverse direction) are compared. The permissible misclosure of average elevations \((d)\) have been calculated by following formulas:

\(\pm 3\text{mm} \sqrt{L} \) - if the average number of the station within 1 km is less than 15,

\(\pm 4\text{mm} \sqrt{L} \) - if the average number of stations within 1 km is greater than 15,

where \( L \) is the length of section in km. For 1 km line mean-square random errors have been calculated by the following formula:

\[
\eta^2 = \frac{1}{4n} \left[ \frac{d^2}{r} \right]
\]

(5)

Where, \( n \) - the number of sections of the line; \( d \) - difference between average elevations of the I and II levelling lines and \( r \) is the section’s length.

For 1 km line mean-square systematic errors calculated by the following formula
\[ \sigma^2 = \frac{1}{4L} \left[ s^2 \right], \]

where \( L \) is the length of line, \( S \) values taken as the difference of a straight line ordinates systematically plotted on a corresponding section of the accumulation differences curve of uniform accumulation.

Then the adjustment of I and II classes levelling networks have been done combined by Local X. Positioning Sistem+ software with the accuracy of I class \( f_{\text{perm.}} = \pm 3 \sqrt{L} \) mm. Where \( L \) is the length of section in km. Hence, as an adjustment result, miclosure and permissible value for each section were calculated. Comparing the separate and combined adjusted results of heights, it found out that heights differences for some marks and benchmarks is up to 1 mm. In order to transform into normal heights systems, the corrections was done in heights differences during the adjustment of I and II classes levelling network.

The accuracy of relative values between points is \( \pm 0.017 - \pm 0.029 \) mGal, and accuracy of absolute values is \( \pm 0.050 - \pm 0.069 \) mGal. Which is 1.4 to 2 times greater than the accuracy required by the technical task and is a high index for similar gravimetric works. The gravimetric connection of the points was determined by the method of gravimetric relative observations, so differences of acceleration of the gravity force between the points was measured. GNY-KB and GNY-KC gravimeters have been used to measure the relative values of gravity force acceleration, the equipment’s square error of a unique measurement does not exceed \( \pm 0.03...0.04 \) mGal.

The effect of atmospheric pressure changes was calculated by this formula

\[ \delta gp = 0.406(\Delta P) \mu \text{Gal} \]  

(7)

Where \( \delta gp \) is the change of the gravity acceleration and \( \Delta P \) is the change of the atmospheric pressure (in millibar units). The maximum changes of the gravity acceleration was 0.8 \( \mu \)Gal, which is considered a minor value and was not taken into account during these observations.

The mean square error of the relative values measurements was estimated by the comparability of the two gravimeters and was calculated by the following formula

\[
\sigma = \pm \sqrt{\frac{\sum_{i=1}^{n} (\Delta g - \bar{\Delta g})^2}{n - 1}},
\]

(8)

Where \( \bar{\Delta g} \) is arithmetic average of relative values, \( \Delta g_i \) the measured value and \( n \) is a number of values. The final accuracy of the measurements was estimated by the average values of the average square errors:

\[
\sigma_3 = \pm \sqrt{\sigma_1^2 + \sigma_2^2},
\]

(9)
Where $\sigma_1$ is the accuracy of the absolute value of the gravity force, $\sigma_2$ is the accuracy of relative values of gravity measurements, $\sigma_3$ is an accuracy of the absolute values of gravity force on benchmarks.

For this purpose, the gravity measurements carried out on benchmarks of I class altitude network by the Institute of Geophysics and Engineering Seismology of the National Academy of Sciences of the RoA [1, 5]. The gravity values (g) of some benchmarks have been obtained by interpolation from gravity map.

As we have not carried out such a huge creation and adjustment work of I and II SAN classes, in 2007 for the control purposes the second time the adjustment have been done by <<Fortran 77>> software based on our database. The software was developed by <<Moscow aero geodetic company>> of Geodesy and Cartography of Federal Agency of the Russian Federation. The program based on a high adjustment algorithm of levelling networks. The I and II classes of SAN adjusted with 3 methods:

1. as an independent network based on 2 initial points,
2. combined
3. least squares method, parametric method taking into account elevation weights.

The third method is accepted, where the mean square error is given for I class $m = 1.7$ mm, for II class $m = 2.2$ mm and after adjustment the results are respectively $m = 1.7$ mm, $m = 2.2$ mm. So, comparing the mean square error of heights between nodal and initial points, it was found that they are in the range of 1-2 cm, and which characterized the high accuracy of the levelling [6]. Comparing this two heights adjusted results obtained by 2 different software’s (the third method of <<Fortran 77>> software and Local X. Positioning System+ ), it was found out that the largest difference is 4.2 mm between 4 marks. Hence, the analyzing the results of two software’s adjustment, it is found out that SGAN of RoA corresponds to the International Geodetic and Cartographic Standards with its density and accuracy. With such network, it is possible to solve any defense, scientific-economic problems.

2 Results and discussion

Satellite technologies allows to get geodetic heights by direct observations in an automated system, so it is necessary to make a relationship between geodetic and normal heights. Necessarily to have geodetic and normal heights on some base points, then create a difference surfaces model, which allows to have geodetic normal heights on the any point of RoA area. So, in an automatic system the geodetic heights (obtained by GPS rover stations) can transfer into normal geodetic heights after some additive corrections, this type of model is called quasigeoid model.

It should be noted that currently astronomical determinations are continued to retain their importance for the determination of vertical astrogeodetic deviations and the creation of the geoid [10, 11, 12].

In order to provide a high accuracy in geoid shape studies, the gravimetric corrections are applied which presented in a article [10]. Egm96, EGM08 are examples of global geoids.

The advantages and disadvantages of each mentioned methods are given. It was found that there are no sufficient data of astrogeodetic and gravimetric measurements in the territory of RA, therefore the application of astrogeodetic and gravimetric methods are not appropriate for the development of the quasigeoid model of RA territory.

Taking into account the dense locations of NGN base points and SGAN benchmarks of RA and the geographical location features of, it is advisable to use the local method of the
In 2008 established the bases for creation methods of quasigeoid: according the existed geodetic materials, tools, financial capacity and professional skills decided to create a quasigeoid model of RoA territory in a local way. Then in the same year technical project was developed to create quasigeoid model of RoA territory in "Geodesy and Cartography Center" SNCO. 185 base points of coordinate catalog of NGN and SGN were taken out, which geodetic and normal heights have been determined using GPS observations and I, II classes levelling. It was selected 333 basic, ground and rock marks from SGAN catalog; the mark’s locations are satisfying to the requirements of GPS monitoring commands. GPS receivers carried out the monitoring in 12-hour sessions. The observed data was converted to RINEX international format for further processing and calculation, then entered into a computer with the measuring data. The heights of quasigeoid have been calculated by following formula using the results of geodetic and normal heights of 185 base points and 333 marks.

\[ \zeta = H - H' \]

where \( H \) – geodetic height, \( H' \)-normal height. The plan of quasigeoid model developed using the digital values data of the plan coordinates of 185 base points, 333 marks and \( \zeta \) quasigeoid heights (Fig. 2). The quasigeoid model of RoA have calculated by Leica Geo office software using linear interpolation method based on 185 base points 320 marks. The quasigeoid model accuracy was checked with 13 marks, and accuracy is 2...8 cm. That marks were not included in calculation before. Within the framework of an interstate agreement, the Swedish specialists have calculated the quasigeoid model of our region using gravity data observed the German gravity satellite (Fig. 3). Then they checked the accuracy of quasigeoid model by the normal heights of our 333 marks. As a result, they obtained the same estimate accuracy as we have. The RoA quasigeoid model accuracy satisfied to the instruction requirements of III, IV classes technical levelling in RoA [3, 4, 12].

![Fig. 2. The locations of observed points (by GPS receivers) for creation quasigeoid model of RA.](image-url)
In 2015 carried out studies of SAT, had been examined 936 marks, about 468 were restored and 84 marks were destroyed. In 2015 technical project was developed for updating data of RoA quasigeoid model. Facts had analyzed and levelling lines had improved in the areas of fewer points. These works carried out in 2015-2017.

**Fig. 3.** A part of the geometric format of a quasigeoid.

The base points have installed, surveying works carried out by GPS receivers and levelers according to the technical project. Then coordinates of base points, geodetic and normal heights were calculated and quasigeoid heights had been calculated by (4) equation [9].

In 2018 some inaccuracies of the quasigeoid model was cleaned, and as a result the points was decreased to 946, at which 631 were newly observed, 28 from <<Spitak geodynamic>> polygon, 20 from <<At Yerevan Close>> geodynamic polygon, 20 from city network, 208 from 2008 created quasigeoid model and 33 base points of NGN and SAN. The observed data entered into the "Leica Geo Office v 8.3" software package.

The quasigeoid model of the whole territory of the RoA created by approximation method and by the differences of the normal and geodetic heights. At first, the new project created and the names of the base points, their geodetic and normal heights, coordinates entered. The base points data which have high accuracy had mentioned as a control points. Then the command was given to calculate heights of quasigeoid and to create a surface. So quasigeoid model made and it bounded with edge’s measured points. In order to obtain the outside heights of that bounded area, alignment and the spread of mode have been done up to required part. During such a spread, the accuracy decreases, because the spread was carried out from the known point to the unknown parts. Depends on a relief and the accuracy of initial points the spread is carried out at a distance not more than from 5 to 15 km.

In an above Fig. 4 is shown a new quasigeoid model for the Republic of Armenia, which computer solution is made by Yunas Angren and Armenian experts. EGM-2008 was localized based on gravimetric balancing of 320 GPS/altitude benchmarks. The approximate accuracy:
• Standard averaged error GPS balanced is about 5 cm,
• The standard accuracy in other locations is 10-20 cm.

Fig. 4. The quasigeoid gravimetric model of the RA territory created by observation data in the Global Satellite Coordinate System.

For the controlling, the new model created by «Trimble Grid Factory v1.41» software using initial point’s data. In this software package the same (previous) data were used, but unlike the previous here the data are entered in the matrix form. Than after creation of model some configurations were made at the boundaries and outside it.

So having models created by two type’s software, their comparison had done to check the differences between two models and their inconsistencies. It was find out that the difference does not exceed 0.001 m [6, 7].

At North and South part of RoA 19 base points were selected to estimate the accuracy of RoA quasigeoid model (The whole territory of Shirak and Lori regions, the northern part of Aragatsotn region). Points chosen such that they should be close to base points and benchmarks of NGN and SAN. The estimated area is 6500 km² (Fig. 5). Normal and geodetic heights had been determined on that selected base points. The observed data sated on the quasigeoid models of RoA 2008 and 2018 update. As a result of data comparison, the accuracy of quasigeoid models was determined according to the selected sections. Data obtained from measurements 2008 and the comparative results of 2018 update models given in Table 1 [2, 8, 10].
For estimating the accuracy of RoA quasigeoid model, the selected area was divided into 5 parts. According to the combined data results of 2018 RoA quasigeoid model, the model accuracy does not exceed 5 cm in Stepanavan area, 6 cm in Gyumri, 5.5 cm in

**Fig. 5.** The scheme of the five selected sections of RA quasigeoid’s territory and its reference points for control measurements purposes.

**Table 1.** Data measurements 2008 and combination results of 2018 quasigeoid models.

<table>
<thead>
<tr>
<th>Point Number</th>
<th>Normal height combined with 2008 quasigeoid model (m)</th>
<th>( \zeta ) (m)</th>
<th>Normal height combined with 2018 quasigeoid model (m)</th>
<th>( \zeta ) (m)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1892.870</td>
<td>0.035</td>
<td>1892.866</td>
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<tr>
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<td>1522.445</td>
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<td>0.077</td>
<td>1518.924</td>
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</tr>
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Spitak, 5 cm in Vanadzor and 5 cm in Aparan-Artik. In table was shown, that the difference between the results were 5…15 cm in case of using the 2008 quasigeoid model and in case of using the 2018 quasigeoid model it doesn’t exceed 6 cm.

Conclusion The update quasigeoid model has high accuracy, which satisfies the instruction requirements of technical leveling works of III, IV classes. It means in cartographic geodetic works, which has such required accuracy to determine the altitudes, instead of geometric or trigonometric levelling the heights of geodetic points will be determined by observations of GPS receivers.

Suggestions Taking into account the current geopolitical situation of the Republic of Armenia, as well as 2.9 points instruction requirements, where is given that all I class lines must be levelling in each 25 years and each 15 years in seismically active areas, and II class lines respectively each 35 and 25 years, it is necessary to make changes in the structure of the RoA altitude network. It means the I class levelling network should be re-observed according to new project, which will provides required geodetic altitudes data for the development of the country's economy.

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