Optimal farm development: mathematical model and method of calculation based on ESIC

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Abstract. The paper develops a mathematical model of the problem of optimization of livestock production, which determines the best development of the farm and the appropriate amount of investment (credit). The performance of the model is shown on the data of the Educational-Scientific-Innovation Center at the Kyrgyz National Agrarian University named after K. I. Skryabin.

1 Introduction

A number of works [1], [2–14] are devoted to the study of private cases, which address the issues of forecasting agricultural production, taking into account exports and imports, and the problem of optimal distribution of investments between farms for the economic development of the region etc.

In this work a mathematical model and method of calculation of the problem of choosing the optimal variant of the farm development have been developed. The model approbation was carried out on the specific data of the farm «Educational Scientific-Innovative Centre» at the ESIC K.I. Skryabin. On the basis of mathematical modelling of production processes, reserves and ways of increasing the volume, quality and efficiency of agricultural production have been revealed, as well as the potential and prospects for development of this farm have been determined.

2 Problem statement

Let a livestock farm with different cultivated areas (irrigated, rain-fed, etc.) in the size of s_k, $k \in K$, plan to develop its activities and needs investment (credit). For the farm, there are several possibilities of producing agricultural products depending on the chosen development option r, $r \in R$. It is considered that the amount of investment (credit) received by the farm depends on the chosen development option. The $r \in R$ option will determine the

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number of animal contents in each production line (milk, meat, etc.) and the size of the area planted for feed crops will be determined.

$$s_k^r(s_k^r = s_k),\ k \in K,\ r \in R$$

3 Objective of the task

It is necessary to determine the best development way of the farm and the corresponding amount of the investment (credit) in such a way that the profit obtained from the production and sale of the product is maximized.

4 Methods

The mathematical model of the problem can be presented as follows.

$$\sum_{j \in J^*} d_j x_j + \sum_{r \in R} (\Delta_l^r - (c^r + \Delta_0^r + k^r(1+\alpha\lambda)/\lambda))y^r \to max \tag{1}$$

under conditions

$$\sum_{r \in R} (a_j^r - \overline{a}_j^r) y^r = x_j,\ j \in J^*, \tag{2}$$

$$\sum_{r \in R} k^r y^r \le G, \tag{3}$$

$$\sum_{r \in R} y^r = 1, \tag{4}$$

$$y^r \in \{0,1\},\quad r \in R, \tag{5}$$

$$x_j \ge 0,\qquad j \in J^*, \tag{6}$$

where

$$\Delta_0^r = \sum_{j \in J_0} \sum_{k \in K} c_{jk}^r \overline{x}_{jk}^r,\qquad \Delta_1^r = \sum_{j \in J_0} c_j \overline{y}_j^r,\quad r \in R,$$

a $\{\overline{x}_{jk}^r\},\quad \{\overline{y}_j^r\},\quad r \in R$ - optimum solutions to the problem

$$L(x,y) = \sum_{j \in J_0} c_j y_j^r - \sum_{k \in K} \sum_{j \in J_0} c_{jk}^r x_{jk}^r \to max,\quad r \in R \tag{7}$$

under conditions

$$\sum_{j \in J_0} x_{jk}^r = s_k^r,\ k \in K,\ r \in R, \tag{8}$$

$$\sum_{k\in K} a_{jk}^{r} x_{jk}^{r} = v_{j}^{r} \geq b_{j}^{r},\ j \in J_0,\ r \in R, \tag{9}$$

$$v_{j}^{r} - b_{j}^{r} = y_{j}^{r},\ j \in J_0,\ r \in R, \tag{10}$$

$$x_{jk}^{r} \geq 0,\ \ j \in J_0,\ k \in K,\ \ r \in R, \tag{11}$$

$$y_{j}^{r} \geq 0,\ j \in J_0,\ r \in R, \tag{12}$$

where *j*- is the index of the type of agricultural produce produced by the farm $j \in J$ and *J* - is the set of indices of the type of agricultural production of a farm, $J=\{1,2,\ldots n\}$.

Consider the set *J* as the union of two disjoint subsets, i.e., $J=J^{*} \cup J_0$, $J^{*} \cap J_0=\emptyset$.

J_0 - is a variety of crop products directed to animal feed, $j \in J_0$;

J^{*} - is a variety of livestock products, $j \in J^{*}$.

Known parameters are:

The size of the sowing area of the *k*-th category used by the farm under the *r*-th development variant (in any variant of $r \in R$ development the farm prefers to use the sowing area of all categories completely, i.e. $s_{k}^{r} = s_{k}),\ k \in K,\ r \in R$;

a_{jk}^{r} - is the yield of *j* -th type of crop on the *k*-th category of sowing area under *r*-th variant development, $j \in J_0,\ k \in K,\ r \in R$;

b_{j}^{r} - is the annual need of the farm for the *j* -type crop production used for animal feed according to *r* -my development option, $j \in J_0,\ r \in R$, where $b_{j}^{r} = \sum_{l\in L} \alpha_{jl}^{r} D_{l}^{r},\ j \in J_0,\ r \in R$;

α_{jl}^{r} - is the annual demand for *j*-type agricultural produce per animal l-breed, based on the productivity and daily feeding diet, according to *r*-th variant of farm development, $j \in J_0,\ l \in L,\ r \in R$;

D_{l}^{r} - is the number of animals of *l*-breed contained in the farm, based on the productivity and daily diet, according to r-mode development, $l \in L,\ r \in R$;

c_j - the wholesale price of the *j*-th type of crop production, $j \in J_0$;

c_{jk}^{r} - is the cost per unit of the size of the *k*-th category of sowing area for *j*-th type of crop in the farm, $j \in J_0,\ k \in K,\ r \in R$;

a_{j}^{r} - is Livestock production, *j* under the *r* variant, where $a_{j}^{r} = \sum_{l\in L} \theta_{jl}^{r} D_{l}^{r},\ j \in J^{*},\ r \in R$;

θ_{jl}^{r} - is the annual volume of *j*-type production obtained from a single *l*-type animal, based on the productivity and daily feeding diet, according to the *r*-th variant of the farm development, $j \in J_0,\ l \in L,\ r \in R$;

k^{r} - size (credit) obtained by the farm under the r-variant development, $r \in R$;

c^{r} - total costs of livestock production under the *r*-th development option, $r \in R$;

d_j - the selling price per unit of the *j*-th type of livestock production, $j \in J^{*}$;

G - is the maximum amount of investment (credit) a farm receives for its development;

Δ_0^r - is the total expenditure of the farm on production of crop production according to the *r*-m variant of development, *r∈R;*

Δ_1^r - is the income received by a farm from the sale of crop production according to *r*-m variant of development, *r∈R.*

λ- is the duration of the loan; α is the interest rate of the loan.

The desired variables are:

x_j - the volume of cattle production of *j*-type to the market, $j \in J^*$;

x_{jk}^r - is the size of the cultivated area of the *k*-th category under the *j*-th type of crop cultivation under the *r*-th variant of farm development, $j \in J_0$, $k \in K$, $r \in R$;

v_j^r - is the production volume of the *j*-type crop production according to the *r*-th development option, $j \in J_0$, $r \in R$;

y_j^r - is the volume of agricultural products of the *j*-th type according to the *r*-th development variant, $j \in J_0$, $r \in R$

y^r - Boolean variable, taking the value 0 or 1, determining the development option, *r∈R.*

Target function (1) defines the maximum net income of a farm from the production and sale of agricultural products;

Limitation (2) requires that livestock production of each type, with the exception of the produce used for the domestic use of the household, shall be equal to the product sold;

Limitation (3) requires that the amount of investment (credit) the farm receives for development should be less than the maximum allowable amount;

Equality (4) states that the farm should choose only one development option;

Condition (5) - the Boolean variable taking the value 0 or 1, (6) requires not the negativity of the desired variables.

Target function (7) defines net income from crop production and marketing;

Limitation (8) requires that the cultivated area of all categories of the farm be fully used to grow crops for feed;

Limitation (9) requires that the production of agricultural produce on the farm should not be less than the need of the farm for domestic use, i.e. for feed.

Equality (10) shows the amount of crop production sold for each type;

Conditions (11) and (12) do not require negative variables.

5 The algorithm for solving the problem

We start the calculations by determining the value of the parameters b_j^r, $j \in J_0$, $r \in R$, a_j^r, $j \in J^*$, $r \in R$. Using the known data s_k^r, $k \in K$, $r \in R$, a_{jk}^r, c_{jk}^r, $j \in J_0$, $k \in K$, $r \in R$, c_j, $j \in J_0$, we calculate the numerical model of the problem according to (7)-(12). By solving the problem for each development variant *r∈R,* we shall determine the optimum plans of the cultivated areas, the allowable by each type of crop $\{\overline{x}_{jk}^r, j \in J_0, k \in K, r \in R\}$ and the corresponding production volumes for each type of crop $\{\overline{v}_j^r, j \in J_0, r \in R\}$, and total consumption $\{\overline{y}_j^r, j \in J_0, k \in K, r \in R\}$, and farm income from crop production respectively, i.e. $\{\Delta_0^r, \Delta_1^r, r \in R\}$.

Next, we formulate a numerical model of the species (1)-(6), using the known data d_j, $j \in J^*$, a_j^r, $\overline{a}_j^r$, $j \in J^*$, $r \in R$, k^r, Δ_0^r, Δ_1^r, c^r, G.

From the solution of the formulated numerical model of type (1)-(6), an efficient development option and corresponding size (credit) are determined, which delivers to the farm the maximum total profit from the production and sale of agricultural products, i.e. the optimum plan $\{\overline{y}^r,\ r \in R\}$ and the corresponding values of the required variables $\{\overline{x}^r_{jk},\ y^r_j,\ j \in J_0,\ k \in K,\ r \in R\}$. The calculation algorithm is ending.

6 Results and discussion

Let's test the performance of the mathematical model and the calculation algorithm based on the ESIC data at KNAU by him. K.I. Skryabin.

The main activity of the ESIC farm is crop and livestock - dairy and meat. As of 2018, there were 47 heads of dairy cows with a yield of 3600 kg, 30 heads of bull and 53 heads of chicks. The area under cultivation is 366 hectares, of which 280 hectares are irrigated and 86 hectares are rain-fed.

In 2018, out of 47 heads of dairy cows, 170437.0 kg of milk was produced, of which 52427 kg was used for domestic needs. Of the 53 heads of cattle with a living mass of 312 kg, 16530 kg was produced, of which 3833 kg was used for internal consumption, 30 heads of bull with a living mass of 235 kg was obtained 7050 kg meat.

As a result, a net profit of 714781.00 soms was received by the farm in 2018 [15, 16].

Furthermore, we calculated 47 head of milk cows, 30 head of bull and 53 chicks according to model (1)-(6), (7)-(12) for the diet of the farm without changing the amount of food needed for animal feed Where the optimal structure of the area planted for feed crops is determined (Table 1). Calculations are made using the MS Excel spreadsheet [17].

Table 1. The optimal structure of the cultivated area on 47 dairy cows with a feeding capacity of 3,600 kg. 30 heads of bull with a living mass of 250 kg, and 53 heads of chick with a living weight of 300 kg for the daily diet of the farm.

Sown area (ha)			Total sown area (ha)
Name of crop	Irrigated area 280 ha	Bogar area 86 ha	
1. Wheat	1.124	43.02	44.144
2. barley	-	42.98	42.98
3 Perennial herbs 3.1. Seno (lucerne)	20.841	-	20.841
3.2. Reception	49.499	-	49.499
3.3. Green feed	47.901	-	47.901
4. Corn 4.1. Silos	38.703	-	38.703
4.2. Grain	121.932	-	121.932
total	280	86	366

An optimum plan for the production of agricultural products for feed and livestock products (Table 2) with the same feeding rations on 47 milk cows with a feeding of 3600 kg of milk, 30 heads of a bull and 53 chicks has been found. The net income of the farm was 2 977 061.08 soms.

Table 2. Optimum plan of production of agricultural products on 47 cows with feeding 3600 kg. 30 heads of bull with a living mass of 250 kg, and 53 heads of chick with a living weight of 300 kg. on the diet of the farm.

Name crop	Production of agricultural products (kg)	domestic needs (kg)	Sales volume products	Price (som/kg)	Farm income (som)	farm consump-tion (som)
1. Wheat	75810.0	75810.0	0	9.70	-	
2. barley	64470.0	64470.0	0	8.40	0	
3. Perennial herbs 3.1 hay	40890.0	40890.0	0	4.50	-	
3.2. haylage	310905.0	310905.0	-	-	-	
3.3 green feed	971438.0	971438.0	-	-	-	
4. corn 4.1 silage	477588.0	477588.0	-	-	-	
4.2. grain	698668.4	16090.0	682578.4	7.00	4778049.0	
amount of income spent					4778049.0	2469862.0
5. livestock						
5.1.milk	170437.0	52427.0	118010.0	20.50	2419205.0	2607686.1
5.2. Beef meat	16530.0	3833.0	12697.0	115.94	1472090.18	993500.0
5.3. breeding bulls	7050.0	-	7050.0	133.30	939765.0	561000.0
Credit payment					-	-
total					9609109.18	6632048.1
Farm profit					2977061.08	-

This is three times the net income received by the farm. This was due to the fact that, in the best case scenario, the unoccupied area was occupied for corn for sale, since the market price of the grain was high, i.e. 7.00 som.

Further, we use the generally accepted standard daily diet of milk cows with a feeding of 3600 kg of milk and a feeding diet of bulls (chicks) for meat with a living weight of 250 (300) kg given in [16].

According to this diet, the annual need for each type of feed per milk cow and per bull (chick) is determined. The annual feeding requirement is calculated for 47 dairy cows, 30 heads of bull and 53 chicks for the household as a whole.

According to the calculated data, the model (7)-(12) determined the optimal structure of the area planted for feed crops for 47 dairy cows, 30 heads of bull and 53 chicks on a daily diet of 3600 kg of milk and beef (chick) 250 (300) kg live weight [16].

An optimal production plan for crop and livestock products also has been defined (Table 3).

Table 3. Optimum production plan for 47 dairy cows with 3600 kg, 30 heads of bull with 250 kg live weight, and 53 heads of chick with 300 kg live weight (with standard feeding diet).

Name crop	Production of agricultural products (kg)	domestic needs (kg)	Sales volume products	Price (som/ kg)	farm income (som)	farm consump-tion (som)
1. Wheat	55295.33	36224.0	19071.33	9.70	184991.9	
2.barley	80210.0	80210.0	0	8.40	0	-
3.Perennial herbs 3.1 hay	78660.0	78660.0	0	4.50	-	-
3.2 haylage	140400.0	140400.0	-	-	-	-
3.3 green feed	78600.0	78600,0	-	-	-	-
4. corn 4.1 silage	159300.0	159300.0	-	-	-	-
4.2 grain	950371.5	25440.5	924931.0	7.00	6474517.0	
amount of income spent					6659509.0	2334899.0
5.livestock						
5.1milk	169200.0	44820.0	124380.0	20.50	2549790.0	2588760.0
5.2 Beef meat	15900.0	-	15900.0	115.94	1843446.0	955590.0
5.3breeding bulls	7500.0	-	7500.0	133.30	999750.0	450750.0
Credit/loan payment					-	-
Total					12052495.0	6329999.0
Farm profit					5722496.0	

Based on the optimal plan of the task (Table 3), the net income of the farm was 5 722 496.00 soms.

Further, according to the standard diet of feeding and feeding requirements we calculate the optimal structure of the sowing area for fodder crops according to model (7) - (12) of 100 dairy cows with feeding of 3600 kg of milk, 30 heads of a bull with live weight 250 kg and 50 calves with a live mass of 300 kg [16-21].

The optimum production plan for crop and livestock production was calculated on the assumption that the farm received a loan of 12 million euros. 6% annual som for 10 years. According to the plan received after payment of 6 per cent of the loan in the amount of 1 920000.00 soms, the net income of the farm amounted to 1 898 796.00 soms (Table 4).

Table 4. Optimal production plan for 100 milk cows with 3600 kg, 30 heads of bull with 250 kg live weight, and 50 heads of chick with 300 kg live weight (with standard feeding).

Name crop	Production of agricultural products (kg)	domestic needs (kg)	Sales volume products	Price (som/ kg)	farm income (som)	farm consumption (som)
1. Wheat	50750.0	50750.0	0	9.70	-	-
2. barley	126950.0	126950.0	0	8.40	0	-
3. Perennial herbs 3.1 hay	115200.0	115200.0	0	4.50	-	-
3.2. haylage	194400.0	194400.0	-	-	-	-
3.3 green feed	1152000.0	1152000.0	-	-	-	-
4. corn 4.1 silage	252000.0	252000.0	-	-	-	-
4.2 grain	531517.4	36500.0	495017.4	7.00	3465122.0	-
amount of income spent.					3465122.0	2019326.0
5.livestock						
5.1milk	360000.0	43200.0	316800.0	20.50	6494400.0	5508000.0
5.2 Beef meat	15000.0	-	15000.0	115.94	1739100.0	901500.0
5.3 breeding bulls	7500.0	-	7500.0	133.30	999750.0	450750.0
Payment for 6% credit for 10 years out of 12 million som					-	1920 000.0
Total					12698372.0	10 799576.0
Farm profit					1 898 796.0	-

7 Conclusions

From the two solutions obtained for the problem according to model (1) - (6), (7) - (12) it follows:

1. For effective development, an ESIC farm must adhere to the generally accepted standard daily feeding ration for 47 dairy cows with a milk yield of 3600 kg of milk and a feeding ration for 30 bull calves and 53 heifers (heifers) for meat with a live weight of 250 (300) kg.

2. Crop production (maize grain) was achieved. 925t; livestock production: milk: 124.4t. Beef Meat - 15.9t. Meat (Beef. Bulls) -7.5t. Net profit - 5.7 million som. These results were obtained without credit.

In another variant of development, when the farm under the standard feeding diet contains in the amount of 100 milk cows with a feeding of 3600 kg milk, 30 heads of a bull and 50 heads of a chick for a meat of 250 (300) kg live weight with a credit of 12 million. For 10 years at 6% per annum it was not an effective development option, since the net income of the farm was 1.89 million. som after payment of 6% of the annual credit in the amount of 1.92 million som.

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