Smoothed Particle Hydrodynamics Simulations of Wave Run-Up over Hexaloc Armour on Rubble-Mound Breakwaters

Muhammad Brilliant Danu Ghazali Hafiz[1*], *Haryo Dwito Armono*[1], and *Muhammad Zikra*[1]

[1]Ocean Engineering, Faculty of Marine Technology, Sepuluh Nopember Institute of Technology, Surabaya, Indonesia

*Corresponding author: brilliantdanu06@gmail.com

Abstract. Indonesian beaches are increasingly vulnerable to abrasion and erosion, requiring reliable protection such as breakwaters. Since crest freeboard determines the level of safety, wave run-up predictions must be accurate. This study evaluates run-up on a 1:2 rubble-mound breakwater slope armored with Hexaloc using SPH (DualSPHysics) to improve design accuracy and efficiency. The geometry was constructed in AutoCAD and SketchUp. The simulation covered wave heights of 0.05–0.13 m, periods of 1.1–1.5 s, and single-layer and double-layer armor configurations. The numerical results were validated against analytical–empirical formulations, particularly the Ahrens relation. Ru/H increases with rising Iribarren number and then tends to plateau at high values. SPH produces Ru/H 0.538–1.076 (single-layer) and 0.588–1.216 (double-layer), while theoretical predictions are higher: 1.616–2.299 and 1.59–2.30. In the test range, single- and double-layer Hexaloc produced comparable run-ups; two layers were equivalent or slightly higher. The SPH–Ahrens fit was highly linear ($R^2 = 0.9999$ single-layer; 0.9948 double-layer), but SPH still predicted lower Ru/H than Ahrens.

1 Introduction

Environmental conditions in coastal areas are influenced by two main factors, namely human activities in coastal areas and natural processes, such as coastal erosion caused by ocean waves. Coastal changes are controlled by sediment transport, longshore currents, wave dynamics, and land-use practices in the surrounding coastal area. When sedimentation occurs, the shoreline shifts seaward, whereas abrasion drives the shoreline landward . These two processes are inherently connected, since erosion at one site may result in accretion at another. Coastal abrasion poses a major problem for Indonesia's shore regions, largely driven by wave forces that gradually wear away the shoreline and push it landward. This situation necessitates the installation of coastal protection structures to ensure shoreline stability and minimize the effects of wave action. To address erosion issues, coastal protection structures such as breakwaters are required. Breakwaters are primarily constructed to attenuate wave energy and provide calmer waters, particularly for harbor protection. For ports, breakwaters are designed to reduce wave agitation within the harbour basin, while also potentially influencing local sedimentation patterns near the entrance channel. In addition, detached/segmented breakwaters have been implemented as coastal defense measures that can modify the nearshore wave climate and consequently influence sediment transport patterns, potentially mitigating shoreline erosion and affecting local accretion/erosion. These structures are typically constructed from stacked concrete units with specific geometric forms, placed on the outermost layer to withstand the highest wave loads. Consequently, their shape and dimensions must be carefully designed to ensure structural stability. Breakwaters may be constructed as extensions of the shoreline to enhance port safety, or positioned offshore as detached structures to help mitigate coastal erosion. Their stability and performance largely depend on how they respond to wave interactions, particularly under extreme wave conditions.

In breakwater design, estimating the wave run-up height is a key parameter because it directly influences structural stability and the safety of the area sheltered behind the structure. Reliable run-up prediction supports a more efficient and robust design, reducing potential risks to both the breakwater and the adjacent coastline. Wave run-up refers to the vertical extension of water level along the surface of a coastal defense structure when impacted by waves [1]. The safety of a breakwater is governed by its crest elevation, which must exceed the predicted run-up height as well as the allowable level of wave overtopping. Wave run-up is defined as the vertical distance between the Still Water Level (SWL) and the highest point on the slope reached by the uprushing water. Wave run-up is governed by the incident wave direction, the geometry of the structure, and the roughness of the slope. This run-up occurs due to momentum transfer when waves impact the breakwater structure.

Numerous physical model studies on wave run-up have been carried out by various researchers at laboratory scale. Irribaren developed a formula to estimate run-up height on armor layers composed of different types of materials. Van der Meer and Stam investigated wave run-up on armor units made of fine and coarse stones, concluding that run-up is influenced by the significant wave height (Hs), structural slope angle (α), structural roughness, and wave period (Tp) [2]. Melito and Melby conducted similar experiments using core-loc armor units, demonstrating that the combined effects of high porosity and friction can reduce run-up height [3]. Schüttrumpf and Van Gent are conducted laboratory test. Laboratory experiments on a 1:6 slope showed that the run-up velocity exceeded the 2% exceedance level $\left(u_{2\%}\right)$ with values of 1.39 for the natural wave spectrum and 1.55 for the Texel Marsen Arsloe (TMA) spectrum. Furthermore, the water layer thickness flowing over the crest of the structure at the 2% exceedance condition $\left(c_{h,2\%}\right)$ was approximately 0.33, highlighting the strong influence of turbulence and aeration on the run-up and overtopping behavior of the breakwater [4].

Pang et al. subsequently carried out a comprehensive review of climate-change-driven coastal erosion using more than 200 global case studies. Their analysis suggests that a projected sea-level rise of 0.3–1.0 m by the end of the century could increase land loss by 20–40% in low-lying deltaic regions [5]. They also evaluated 95 case studies focusing on hydrodynamics and shoreline erosion worldwide, revealing that short-period waves (5–8 s) are capable of generating substantial sediment transport, leading to coastal retreat exceeding 5 m/year in highly energetic environments. The study highlights a methodological comparison: field measurements yield realistic data but are expensive and spatially limited, whereas numerical modeling offers greater flexibility but depends heavily on robust validation. Overall, the findings emphasize the need to integrate numerical simulations with field observations in coastal morphodynamic research.

Regarding coastal protection, geometric design criteria for berm breakwaters have been established based on a series of laboratory experiments. These tests yielded empirical formulas for assessing the stability of armour layers and indicated that berm breakwaters with milder slopes offer greater resistance to high wave run-up. Nonetheless, the findings are constrained by the laboratory scale, so site-specific calibration is still required before field implementation. Subsequently, numerical investigations were carried out using the Smoothed Particle Hydrodynamics (SPH) approach. DualSPHysics was employed to simulate wave overtopping on rubble-mound breakwaters, and the results showed that the SPH model can reproduce overtopping discharges with discrepancies of less than 15% relative to experimental data, especially under high-wave conditions and for permeable structures [6]. A key drawback, however, is the substantial computational cost when applied to large spatial domains. Wave run-up on breakwaters has also been analysed using a CFD-based RANS model. The principal outcome of this study was a validation curve comparing numerical predictions with laboratory measurements, revealing an average error of less than 10% for run-up heights on structures with a 1:2 slope. The results confirm the strength of RANS in representing turbulent processes, while also highlighting its limitations in accurately resolving extreme overtopping events with large runoff volumes.

The main aim of this study is to investigate wave run-up on sloped breakwaters using numerical modelling. The breakwater geometry is generated in SketchUp and simulated in DualSPHysics to examine hydrodynamic behaviour under varying wave and structural configurations. Key parameters analysed include wave height, wave period, water level conditions, and the slope angle of the breakwater. The outcomes of this work are expected to enhance understanding of how these wave and structural factors influence breakwater performance. Practically, the insights gained will offer coastal engineers and planners more reliable guidance for designing breakwaters that maintain stability and safety while keeping construction costs efficient. Moreover, by comparing the results with findings from earlier studies, this research underscores both the strengths and the limitations of SPH-based modelling for analysing coastal structures.

2 Methods

This research examines wave run-up on rubble-mound coastal structures equipped with Hexaloc armour units under irregular wave conditions. The study utilizes data gathered from literature reviews and digital geometric models developed in AutoCAD and SketchUp, which were subsequently imported into the DualSPHysics framework for numerical simulation.

2.1 Hexaloc Armour Unit

In coastal defense systems, one of the key elements is the armour rock layer. This outer layer serves to dissipate wave energy before it reaches the shore. These protective rocks, commonly referred to as armor units, are available in various forms such as dolos, tetrapods, A-jacks, and others. Typically cast from concrete, armor units are designed with specific geometries that promote stronger interlocking compared to conventional rock, thereby improving the overall stability of the structure.

Several countries have developed artificial armor units as substitutes for natural stones in coastal protection systems, featuring diverse shapes and stability characteristics. Alongside Europe and the United States, Indonesia has introduced its own artificial armor unit known as Hexaloc. This unit resembles the A-Jack and hexapod forms. The hexapod was first introduced in the United States in 1959, while the A-Jacks were developed later in 1998. Both units feature legs with tapered circular profiles. The A-Jacks are created by combining two identical components into one structure, resulting in six legs extending from a central filleted hub, each with a square cross-section. In contrast, the Hexaloc has six legs with a hexagonal cross-section that meet at a single joint without a filleted center [7]. Illustrations of these units are provided in Figure 1 and Table 1.

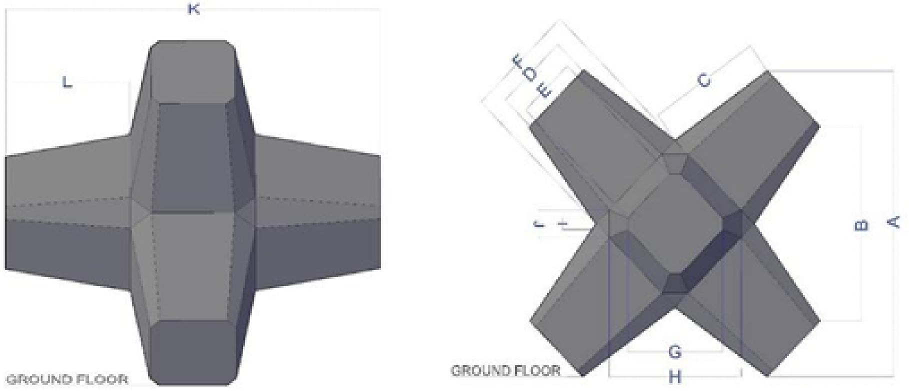

Fig. 1. Hexaloc

Table 1. Hexaloc Dimension

Symbol	A	B	C	D	E	F	G	H	I	J	K	L
Length (mm)	2106	1340	823	542	429	812	687	961	79	191	2437	808

2.2 Run-up and Irribaren Number

When waves strike a structure, their momentum drives the water to travel up along the surface of the structure. The maximum vertical elevation reached by this uprush is termed the wave run-up. This parameter is critical in the design of coastal protection works because it governs the required crest elevation of the structure. Wave run-up is controlled by several factors, including the geometry and slope of the structure, surface roughness, local water depth at the toe, and the incident wave characteristics. Run-up is commonly expressed in two forms: the mean run-up, defined as the average uprush height of all waves, and the 2% run-up, defined as the average of the highest 2% of run-up values from a sample of 100 waves [8]. The type of breaking that occurs can be classified using the Iribarren number (ξ), also known as the surf similarity parameter. Below are the equations used to calculate wave run-up.

$$\xi = \frac{\tan tan\, \alpha}{\sqrt{\frac{2\pi H_s}{gT^2}}} \qquad (1)$$

where, ξ is the Irribaren number, α is the structure slope angle, H_s is the significant wave height, g is Earth's gravity, which has a value of 9.8 m/s^2, and T is the wave period.

Several studies have introduced empirical formulas for predicting wave run-up. Based on laboratory tests involving irregular waves on smooth, impermeable slopes, Mase developed an empirical relationship for estimating maximum run-up, valid for slope gradients between 1/30 and 1/5 and wave steepness values ranging from 0.007 to 0.07 [9]. The following formula is derived from the study.

$$\frac{R_{max}}{H_0} = 2.2\xi_0^{0.77} \tag{2}$$

Douglass examined wave run-up by eliminating the influence of beach slope from the governing equation. He reasoned that determining the slope is challenging and has only a minor effect on run-up, allowing the tan θ term in the Iribarren number to be substituted with a constant value [10]. The equation for calculating wave run-up is as follows:

$$\frac{R_{max}}{H_0} = \frac{0.12}{\sqrt{\frac{H_0}{L_0}}} \tag{3}$$

$$L_0 = \frac{gT^2}{2\pi} \tag{4}$$

where, R_{max} is the run-up maximum, L_0 is wave length, and H_0 is wave height.

Ahrens and Heimbaugh subsequently formulated an upper-limit equation to estimate the maximum run-up of irregular waves on riprap revetments, which represent rough and permeable slopes. Their formulation was derived from laboratory experiments conducted at a 1:16 scale [11]. In their study, run-up was defined as the elevation reached by green water only (excluding spray), and the resulting prediction is expressed as:

$$\frac{R_{max}}{H_{mo}} = \frac{1.022\xi}{1+0.247\xi} \tag{5}$$

2.3 Smoothed Particle Hydrodynamic

Gomez-Gesteira et al. characterize Smoothed Particle Hydrodynamics (SPH) as a meshless Lagrangian technique that has been widely adopted in Computational Fluid Dynamics (CFD) [12]. In SPH, the fluid is represented by discrete particles that can interact with solid boundaries and naturally follow large deformations in domains with moving boundaries. Over the years, continuous improvements to the SPH formulation have led to satisfactory levels of accuracy, stability, and robustness for engineering applications. The fluid field is discretized into a cloud of numerical particles, making SPH well suited for modelling a broad spectrum of problems, such as astrophysical flows, free-surface dynamics, and complex mixing processes. Unlike conventional grid-based CFD approaches, SPH does not require a fixed mesh, which offers certain computational advantages. In this framework, continuous media are represented by ensembles of particles, and for fluid flow, the Navier–Stokes equations are evaluated at each particle position based on interactions with neighboring particles. These neighbors are identified through a kernel function with a specified smoothing length, defined over a circular support in two dimensions or a spherical support in three dimensions. At every timestep, the physical variables of all particles are updated, and the particles are advected according to their newly computed properties.

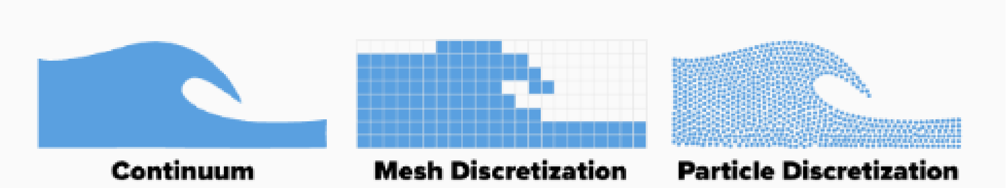

Fig. 2. The Difference Between Mesh and Particle Discretization of Continous Free-Surface Flow [13]

2.4 DualSPHysics

DualSPHysics is an open-source computational tool designed specifically for modeling free-surface flow processes. It utilizes the Smoothed Particle Hydrodynamics (SPH) approach and incorporates hardware acceleration to efficiently compute particle motion within the simulation domain. Developed for real engineering applications, DualSPHysics extends the capabilities of the SPHysics model and is written in C++ with CUDA optimization. The SPH methodology in DualSPHysics is organized into three main stages: Neighbour List (NL), Particle Interaction (PI), and System Update (SU) [14]. To generate numerical outputs, users must proceed through a structured sequence of steps provided within the software. Workflow diagram illustrated in Figure 3.

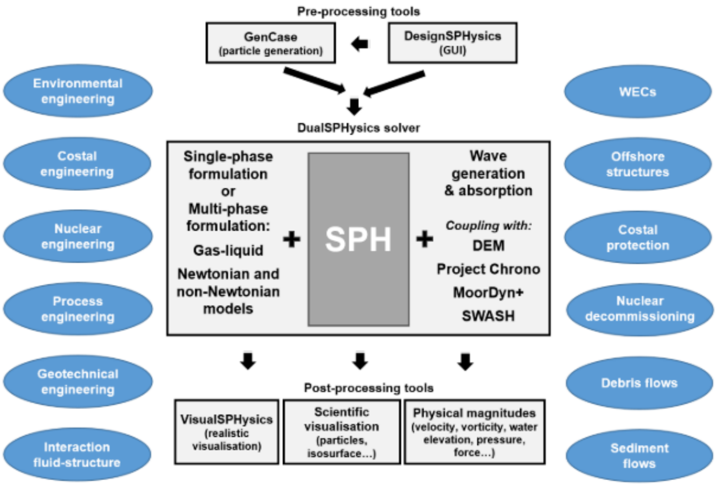

Fig. 3. Workflow of The DualSPHysics Program [14]

2.5 Data Analysis

In this study, the authors employed a breakwater with a length of 1.75 m, a height of 0.70 m, and a width of 2 m, placed at a water depth of 0.65 m. In this study, the breakwater slope was fixed at 1:2. The slope was not treated as a decision variable; instead, it was adopted as the baseline geometry of the investigated breakwater model. Accordingly, the analysis focuses on the effects of wave conditions and armour-unit arrangement on wave run-up for this fixed slope. Three variables were varied in the experiments, namely wave height, wave period, and the number of structural arrangements. Two wave gauges (WG1 and WG2) were used to monitor the free surface elevation. WG1 was located in the constant depth section downstream of the wave generator to record the generated wave conditions prior to the interaction with the structure. WG2 was positioned at the breakwater face to capture the near structure water surface evolution used for run-up estimation. Wave propagation was from left to right, from the wave generator toward the breakwater. For further details, see Figure 4 and Table 2.

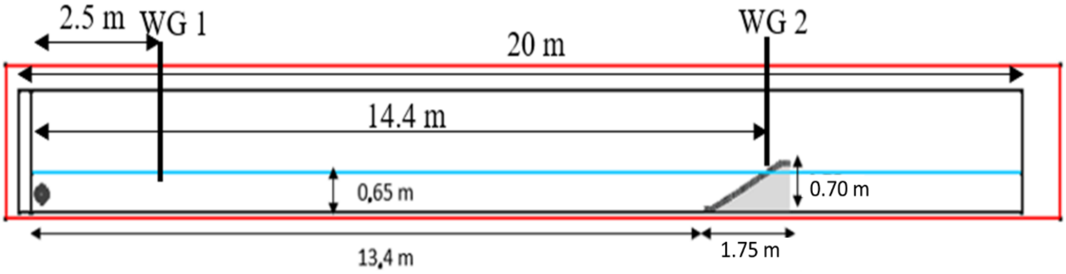

Fig. 4. Test Set-up in DualSPHysics

Table 2. Test Scenario

Variations	Number Of Layers	Wave Height (m)	Wave Period (s)
1	1	0.05	1.5
2		0.07	1.2
3		0.09	1.2
4		0.11	1.1
5		0.13	1.1
6	2	0.05	1.5
7		0.07	1.2
8		0.09	1.2
9		0.11	1.1
10		0.13	1.1

In this study, calibration and validation were carried out by comparing numerical results for regular waves with analytical solutions derived from second-order wave theory. DualSPHysics applies the SPH method using discrete particles, where the distance particle (dp) can be tuned to control numerical resolution: a smaller dp increases accuracy but also raises computational demand. Considering hardware limitations, a value of dp = 0.03 m was adopted. Each simulation was run for 100 s, with wave gauges placed at x = 2.5 m and x = 14.4 m from the piston. The calibration and validation results, presented in Figure 5, show good correspondence with theory and support the use of this numerical wave tank for further simulations. The deviation between numerical and analytical results, quantified using the root-mean-square error (RMSE), is on the order of 0.07703.

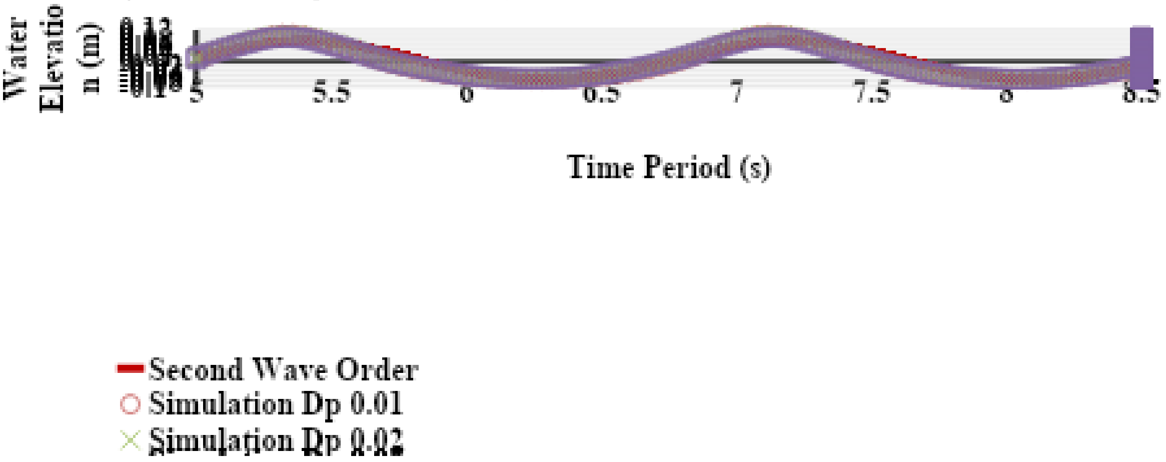

Fig. 5. DualSPHysics Model Validation Based on Second Wave Order Theory

3 Result and Discussion

3.1 Wave Run-up Based on Mase's Theory, Douglass's Theory, and Ahren's Theory

The evaluation of wave run-up on the non-overtopping breakwater in this study utilizes the formulations proposed by Mase, Douglass, and Ahrens, as presented in Eq. (2), Eq.(3), and Eq. (5). These models are selected because they align most closely with the conditions examined in this research. The resulting calculations serve as the foundation for validating wave run-up predictions generated using the Smoothed Particle Hydrodynamics (SPH) method. An example computation is included, beginning with the estimation of the wavelength (L) using Eq. (4) and the Iribarren number using Eq. (1). Table 3 presents the computed wave run-up values for each variation tested on the breakwater structure.

Table 3. Wave Run-up Height (m) Based on Theory

Variation	Wave Height, H (m)	Wave Height Significant, Hs (m)	Wave Period, T (s)	Wave Length, L (m)	Iribarren Number, ξ	*Ru*/H		
						Mase (1989)	Douglass (1992)	Ahrens (1988)
1	0.05	0.061	1.5	3.514	5.061	7.668	0.916	2.299
2	0.07	0.070	1.2	2.249	3.781	6.126	0.680	1.998
3	0.09	0.088	1.2	2.249	3.378	5.616	0.607	1.882
4	0.11	0.106	1.1	1.890	2.811	4.876	0.505	1.696
5	0.13	0.125	1.1	1.890	2.593	4.582	0.466	1.616
6	0.05	0.061	1.5	3.514	5.067	7.675	0.911	2.300
7	0.07	0.074	1.2	2.249	3.683	6.003	0.662	1.971
8	0.09	0.090	1.2	2.249	3.330	5.555	0.599	1.868
9	0.11	0.112	1.1	1.890	2.745	4.788	0.493	1.672
10	0.13	0.132	1.1	1.890	2.528	4.494	0.454	1.591

3.2 Wave Run-up Based on SPH Method

This research utilizes a numerical model based on the Smoothed Particle Hydrodynamics (SPH) method as implemented in DualSPHysics. As an example, a simulation output of wave run-up height is presented. The run-up is quantified as the maximum vertical position attained by free-surface particles when incident waves interact with the breakwater. The particle coordinates in the x, y, and z directions are illustrated in Figure 6. Consistent with the standard definition of wave run-up, the reported values are referenced to the still-water level, obtained by subtracting the initial water depth from the maximum free-surface elevation. The corresponding run-up values for each tested configuration of the breakwater are summarized in Table 4.

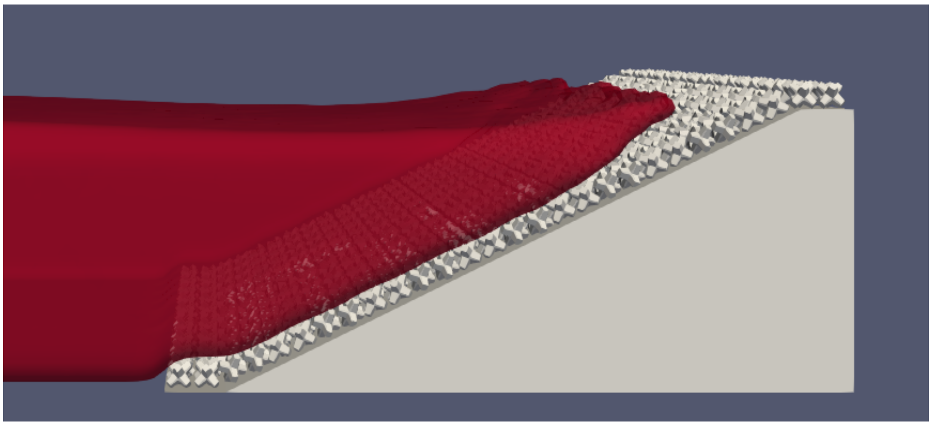

Fig. 6. Wave Run-up Height on Breakwater

Table 4. Wave Run-up Height (m) Based on SPH Method

Variation	Wave Height, H (m)	Wave Height Significant, Hs (m)	Wave Period, T (s)	Wave Length, L (m)	Iribarren Number, ξ	Wave Run-up, Ru (m)	*Ru*/H
1	0.05	0.061	1.5	3.514	5.061	0.066	1.076
2	0.07	0.070	1.2	2.249	3.781	0.062	0.891
3	0.09	0.088	1.2	2.249	3.378	0.069	0.783
4	0.11	0.106	1.1	1.890	2.811	0.066	0.618
5	0.13	0.125	1.1	1.890	2.593	0.067	0.538
6	0.05	0.061	1.5	3.514	5.067	0.074	1.216
7	0.07	0.074	1.2	2.249	3.683	0.068	0.925
8	0.09	0.090	1.2	2.249	3.330	0.079	0.872
9	0.11	0.112	1.1	1.890	2.745	0.078	0.696
10	0.13	0.132	1.1	1.890	2.528	0.077	0.588

3.3 Validation Wave Run-up Based on Previous Theory

In this section, the numerical results produced by the Smoothed Particle Hydrodynamics (SPH) model in DualSPHysics are compared with findings from earlier studies for validation purposes. Figure 7 presents the computed wave run-up (Ru/H) plotted against the Iribarren number, alongside data reported in previous research. In this plot, the green curve representing Ahrens' riprap experiments is used as the primary benchmark to assess the accuracy and consistency of the SPH-based run-up predictions in the present study.

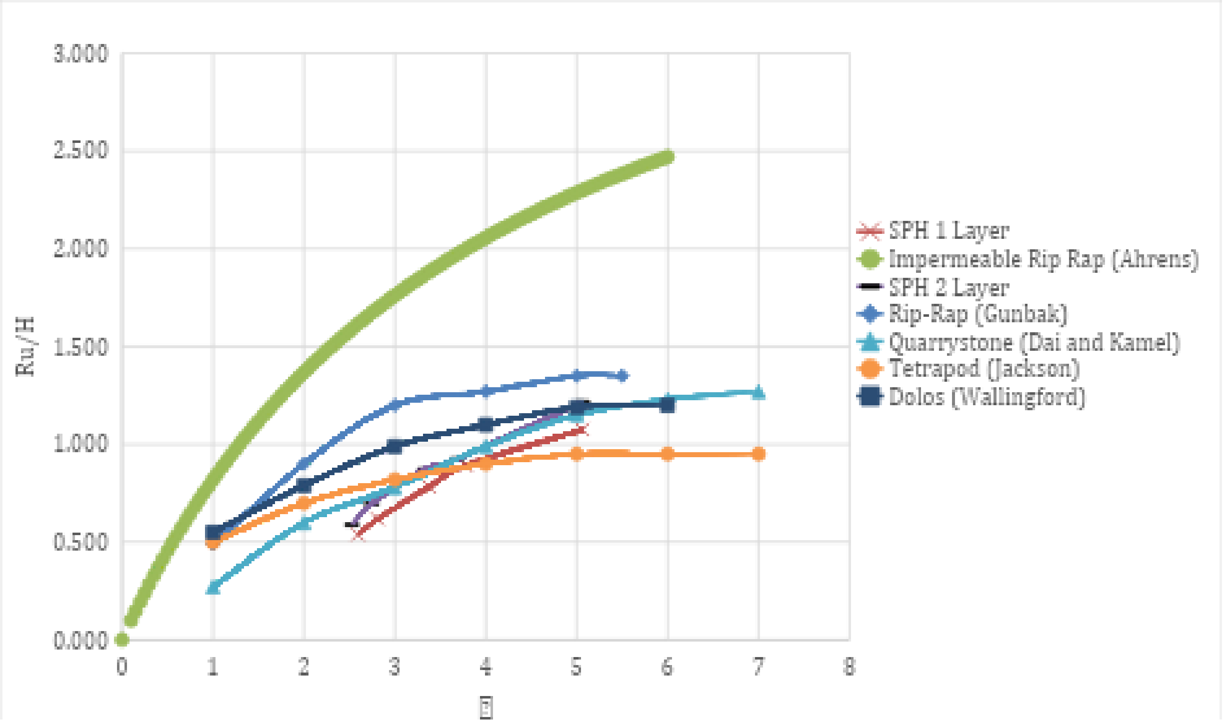

Fig. 7. The Influence of the Iribarren Number on the Wave Run-up

The graph illustrates the relationship between the non-dimensional wave run-up (Ru/H) and the Iribarren number (ξ) for various types of armour layers, together with numerical predictions from the SPH simulations for one-layer and two-layer Hexaloc configurations. Overall, all curves exhibit an increasing trend with ξ, consistent with classical run-up theory for rough and porous slopes. The Ahrens curve for impermeable riprap lies significantly above all other datasets, indicating that rough yet non-porous slopes tend to produce higher maximum run-up due to reduced energy dissipation. In contrast, concrete armour units such as tetrapods (Jackson) and dolos (Wallingford) display lower and more stable Ru/H values for ξ > 4, reflecting the enhanced dissipation caused by interlocking and higher porosity [15].

The SPH results both the single and double layer configurations generally fall below the riprap datasets of Gunbak and the quarry stone data of Dai & Kamel, suggesting that the Hexaloc armour in SPH simulations dissipates wave energy more effectively than natural stone armours. The differences between the one layer and two layer SPH results are minor, implying that adding a second layer does not significantly increase run-up, likely because internal permeability remains high enough to dissipate wave energy before reaching the upper slope. Overall, the graph demonstrates that SPH predictions fall within a physically consistent range compared with existing experimental studies. However, the systematic underestimation relative to the Ahrens upper bound formulation at high ξ values is notable and reflects known SPH behaviour, particularly the tendency for numerical over damping and sensitivity to particle resolution.

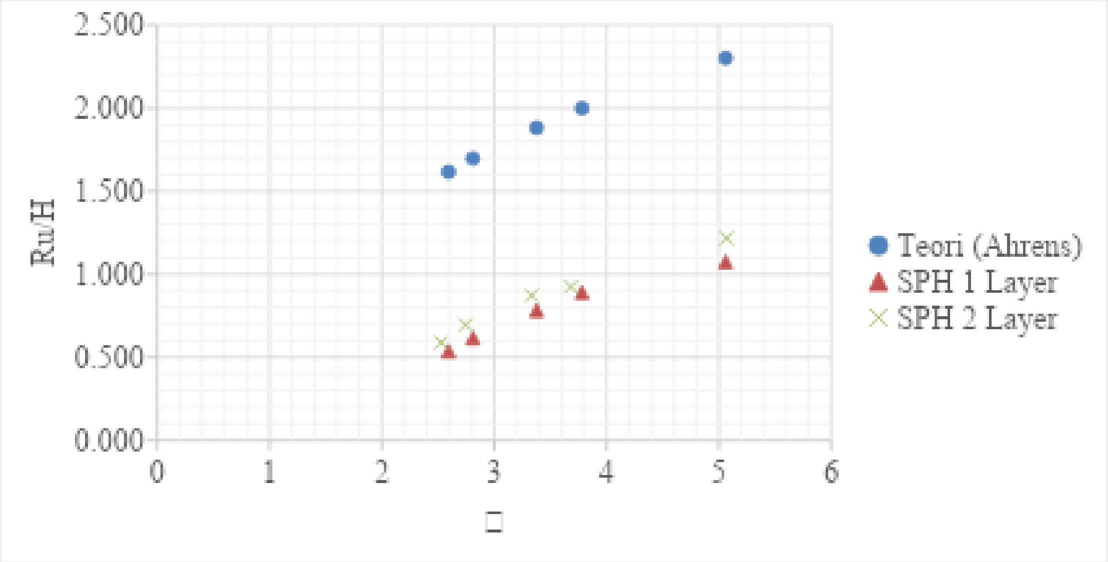

Fig. 8. The Relationship Between Wave Run-up Characteristic and The Iribarren Number

This graph compares the non-dimensional wave run-up (Ru/H) obtained from the SPH simulations with Ahrens' theoretical upper-bound curve for riprap revetments. The Ahrens data (blue markers) consistently exhibit higher run-up values across all Iribarren numbers (ξ), representing the behaviour of rough, low-permeability stone slopes. In contrast, the SPH results for both the one-layer and two-layer Hexaloc configurations are noticeably lower, although they follow a similar upward trend as ξ increases. This indicates that the numerical model successfully reproduces the fundamental dependence of run-up on the surf similarity parameter.

The separation between the SPH curves and the Ahrens reference highlights the stronger energy dissipation associated with Hexaloc armour, which has greater porosity and interlocking compared to conventional riprap. Moreover, the small difference between the one-layer and two-layer simulations suggests that adding an additional armour layer does not significantly alter the run-up response, likely because the overall permeability and dissipation characteristics of the structure remain similar. Overall, the figure demonstrates that the SPH model captures the qualitative behaviour of wave run-up while reflecting the unique hydrodynamic performance of Hexaloc armour relative to classical riprap systems.

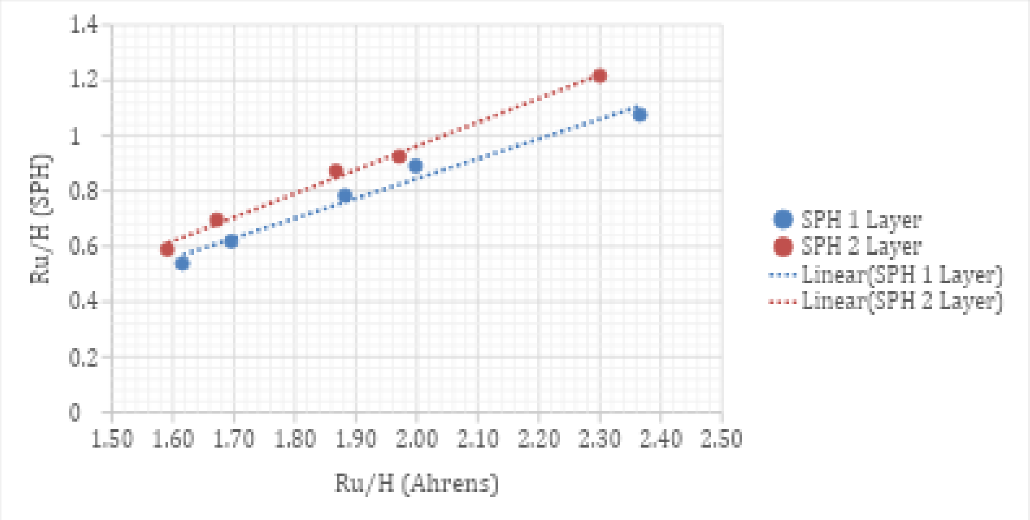

Fig. 9. Run-up Relationship Between SPH and Ahrens Theory

Figure 9 compares SPH-predicted run-up (Ru/H) with the theoretical values from Ahrens. Both the single-layer and double-layer Hexaloc configurations show a strong linear correlation with the Ahrens curve ($R^2 > 0.97$), indicating that the SPH model reproduces the same trend. However, the regression lines lie below the 1:1 line, confirming that SPH systematically predicts lower run-up than the Ahrens upper bound, with the two-layer configuration yielding slightly higher values than the single layer.

4 Conclusion

The comparison between SPH simulations and Ahrens' upper-bound formulation shows that the DualSPHysics model reproduces the expected increase of non-dimensional run-up (Ru/H) with the Iribarren number. Regression analysis between Ru/H(SPH) and Ru/H(Ahrens) yields very high coefficients of determination ($R^2 > 0.97$) for both single- and double-layer Hexaloc configurations, indicating that the SPH results are strongly correlated with the classical riprap run-up theory and capture its overall trend.

Despite this strong correlation, the SPH-predicted run-up values are consistently lower than Ahrens' theoretical curve, reflecting the higher porosity and energy dissipation capacity of Hexaloc armour compared with the impermeable riprap used in the reference formulation. The two-layer Hexaloc configuration produces slightly higher run-up than the single-layer case, but the difference is modest, suggesting that the additional layer does not fundamentally change the hydraulic response because the overall permeability of the armour system remains high.

These findings imply that SPH-based modelling with DualSPHysics is suitable for analysing relative trends in wave run-up and for comparing alternative armour configurations. However, for use in design practice, the systematic offset from Ahrens' upper bound indicates that case-specific calibration against laboratory or field data is still required to obtain conservative crest elevations for rubble-mound structures armoured with Hexaloc units.

References

[1] B. Triatmodjo, Teknik Pantai. Yogyakarta, Indonesia: Beta Offset, 1999.

[2] J. W. van der Meer and C. M. Stam, "Wave run-up on smooth and rock slopes of coastal structures," Ocean Engineering, 1992.

[3] I. Melito and J. A. Melby, "Wave runup, transmission, and reflection for structures armored with CORE LOC®," Coastal Engineering, vol. 45, no. 1, pp. 33–52, 2002, doi: 10.1016/S0378-3839(01)00044-8.

[4] H. Schüttrumpf and M. R. A. Van Gent, "Wave overtopping at seadikes," Coastal Structures 2003 - Proceedings of the Conference, vol. 40733, no. June, pp. 431–443, 2003, doi: 10.1061/40733(147)36.

[5] T. Pang, X. Wang, R. A. Nawaz, G. Keefe, and T. Adekanmbi, “Coastal erosion and climate change: A review on coastal-change process and modeling,” Ambio, vol. 52, no. 12, pp. 2034–2052, 2023, doi: 10.1007/s13280 023-01901-9.

[6] A. J. C. Crespo et al., “DualSPHysics: Open-source parallel CFD solver based on Smoothed Particle Hydrodynamics (SPH),” Comput Phys Commun, vol. 187, pp. 204–216, Feb. 2015, doi: 10.1016/j.cpc.2014.10.004.

[7] C. Irawan and H. D. Armono, “Numerical Study of the of Proposed Hexaloc Concrete Armor Units under Static Loading,” IOP Conf Ser Earth Environ Sci, vol. 1250, no. 1, 2023, doi: 10.1088/1755 1315/1250/1/012005.

[8] J. P. Ahrens, “Irregular wave runup on smooth slopes.,” U.S. Army Coastal Engineering Research Center, Coastal Engineering Technical Aid, vol. 81–17, 1981.

[9] H. Mase, “Random Wave Runup Height on Gentle Slope,” J Waterw Port Coast Ocean Eng, vol. 115, no. 5, pp. 649–661, 1989, doi: 10.1061/(asce)0733-950x(1989)115:5(649).

[10] S. L. Douglass, “Estimating extreme values of run-up on beaches,” Journal of Waterway, Port, Coastal, and Ocean Engineering, vol. 118, no. 2, pp. 220–224, 1992.

[11] J. P. Ahrens, “Approximate upper limit of irregular wave runup on riprap,” Journal of Waterway, Port, Coastal, and Ocean Engineering, vol. 114, no. 4, pp. 614–635, 1988.

[12] G. Argente, M. E. Gómez-Martín, and J. R. Medina, “Hydraulic stability of the armor layer of overtopped breakwaters,” J Mar Sci Eng, vol. 6, no. 4, pp. 1–13, 2018, doi: 10.3390/jmse6040143.

[13] Ferryanto, O. Setyandito, Nizam, N. Yuwono, and W. Trihadi, “Wave run-up characteristic on armour layer breakwater using the smoothed particle hydrodynamics (SPH) method,” in IOP Conference Series: Earth and Environmental Science, Institute of Physics, 2024. doi: 10.1088/1755-1315/1324/1/012031.

[14] J. M. Domínguez et al., “State-of-the-art SPH solver DualSPHysics: from fluid dynamics to multiphysics problems,” pp. 1–43, 2021, doi: 10.1007/s40571-021-00404-2.

[15] M. A. Losada and L. A. Giménez-Curto, Mound Breakwaters Under Wave Attack. Santander, Spain: Dept. of Oceanographical and Ports Engineering, University of Santander, 1979.